Chapter 7: Motion in Two Dimensions

Practice Problems

page 136

1. A stone is thrown horizontally at a speed of +5.0 m/s from the top of a cliff 78.4 m high.

   a. How long does it take the stone to reach the bottom of the cliff?

   Since $v_y = 0$, $y = v_y t + \frac{1}{2}gt^2$ becomes

   $y = \frac{1}{2}gt^2$, or

   $t^2 = \frac{2y}{g} = \frac{2(-78.4\text{ m})}{-9.80\text{ m/s}^2} = 16.0\text{ s}^2$,

   $t = \sqrt{16.05^2} = 4.00\text{ s}$.

   b. How far from the base of the cliff does the stone strike the ground?

   $x = v_x t = (5.0\text{ m/s})(4.00\text{ s}) = 20\text{ m}$

   c. What are the horizontal and vertical components of the velocity of the stone just before it hits the ground?

   $v_x = 5.0$ m/s. This is the same as the initial horizontal speed because the acceleration of gravity influences only the vertical motion. For the vertical component, use $v_f = v_i + gt$ with $v_f = v_y$ and v_i, the initial vertical component of velocity zero. At $t = 4.00$ s,
   $v_y = gt = (-9.80\text{ m/s}^2)(4.00\text{ s})$
   $= -39.2\text{ m/s}$.

2. How would the three answers to Problem 1 change if

   a. the stone were thrown with twice the horizontal speed?

   (a) no change; 4.00 s

   (b) twice the previous distance; 40 m

   (c) v_x doubles; 10 m/s
   no change in v_y; -39.2 m/s

Practice Problems

   b. the stone were thrown with the same speed but the cliff was twice as high?

   (a) increases by $\sqrt{2}$, since $t = \sqrt{\frac{2y}{g}}$ and y doubles; 5.66 s

   (b) increases by $\sqrt{2}$, since t increases by $\sqrt{2}$; 28 m

   (c) no change in v_x; 5.0 m/s
   v_y increases by $\sqrt{2}$, since t increases by $\sqrt{2}$; -55.4 m/s

3. A steel ball rolls with constant velocity across a tabletop 0.950 m high. It rolls off and hits the ground +0.352 m horizontally from the edge of the table. How fast was the ball rolling?

   Since $v_y = 0$, $y = \frac{1}{2}gt^2$ and the time to reach the ground is

   $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(-0.950\text{ m})}{-9.80\text{ m/s}}} = 0.440\text{ s}$.

   From $x = v_x t$,

   $v_x = \frac{x}{t} = \frac{0.352\text{ m}}{0.440\text{ s}} = 0.800\text{ m/s}$.

page 137

4. An auto, moving too fast on a horizontal stretch of mountain road, slides off the road, falling into deep snow 43.9 m below the road and 87.7 m beyond the edge of the road.

   a. How long did the auto take to fall?

   Following the method of Practice Problem 3,

   $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(-43.9\text{ m})}{-9.80\text{ m/s}^2}} = 2.99\text{ s}$.

b. How fast was it going when it left the road? (in m/s and km/h)

$$v_x = \frac{x}{t} = \frac{87.7\ \text{m}}{2.99\ \text{s}} = 29.3\ \text{m/s}$$

$$= 29.3\ \text{m/s}\left[\frac{1\ \text{km}}{1000\ \text{m}}\right]\left[\frac{3600\ \text{s}}{1\ \text{h}}\right]$$

$$= 105\ \text{km/h}.$$

c. What was its acceleration 10 m below the edge of the road?

$g = -9.80\ \text{m/s}^2$ at at all times.

page 139

5. A player kicks a football from ground level with a velocity of magnitude 27.0 m/s at an angle of 30.0° above the horizontal, Figure 7–6. Find

a. its "hang time," that is, the time the ball is in the air.

$$v_x = v_i \cos\theta = (27.0\ \text{m/s}) \cos 30.0^\circ = 23.4\ \text{m/s}$$

$$v_y = v_i \sin\theta = (27.0\ \text{m/s}) \sin 30.0^\circ = 13.5\ \text{m/s}$$

When it lands, $y = v_y t + \frac{1}{2}gt^2 = 0$.

Therefore,

$$t = \frac{-2v_y}{g} = \frac{-2(13.5\ \text{m/s})}{-9.8\ \text{m/s}^2} = 2.76\ \text{s}.$$

b. the distance the ball travels before it hits the ground.

$$x = v_x t = (23.4\ \text{m/s})(2.76\ \text{s}) = 64.6\ \text{m}$$

c. its maximum height.

Maximum height occurs at half the "long time," or 1.38 s. Thus,

$$y = v_y t + \frac{1}{2}gt^2$$

$$= (13.5\ \text{m/s})(1.38\ \text{s})$$

$$+ \frac{1}{2}(-9.80\ \text{m/s}^2)(1.38\ \text{s})^2$$

$$= 18.6\ \text{m} - 9.3\ \text{m} = 9.3\ \text{m}.$$

Practice Problems

6. The kicker now kicks the ball with the same speed, but at 60.0° from the horizontal or 30.0° from the vertical. Find

a. its "hang time," that is, the time the ball is in the air.

Following the method of Practice Problem 5,

$$v_x = v_i \cos\theta = (27.0\ \text{m/s})\cos 60.0^\circ = 13.5\ \text{m/s}$$

$$v_y = v_i \sin\theta = (27.0\ \text{m/s})\sin 60.0^\circ = 23.4\ \text{m/s}$$

$$t = \frac{-2v_y}{g} = \frac{-2(23.4\ \text{m/s})}{-9.80\ \text{m/s}^2} = 4.78\ \text{s}.$$

b. the distance the ball travels before it hits the ground.

$$x = v_x t = (13.5\ \text{m/s})(4.78\ \text{s}) = 64.5\ \text{m}.$$

c. its maximum height.

at $t = \frac{1}{2}(4.78\ \text{s}) = 2.39\ \text{s}$,

$$y = v_y t + \frac{1}{2}gt^2$$

$$= (23.4\ \text{m/s})(2.39\ \text{s})$$

$$+ \frac{1}{2}(-9.80\ \text{m/s}^2)(2.39\ \text{s})^2 = 27.9\ \text{m}.$$

7. Using the results for Practice Problems 5 and 6, compare qualitatively the flight times, ranges, and maximum heights for projectiles launched with high and low trajectories, when the high angle is the complement of the low angle. (The complement of angle θ is $(90^\circ - \theta)$.)

For two projectiles with the same initial velocity and complementary launch angles, the projectile with the higher trajectory has the longeest flight time and the greatest maximum height. The ranges are the same.

7

Practice Problems

8. A rude tourist throws a peach pit horizontally with a 7.0 m/s velocity out of an elevator cage.

a. If the elevator is not moving, how long will the pit take to reach the ground, 17.0 m below?

$v_y = 0$ and $y = -17.0$ m = ground level.

Therefore, $y = v_y t = \frac{1}{2}gt^2$ becomes

$$y = \frac{1}{2}gt^2 \text{ or } t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(-17.0\text{ m})}{-9.80\text{ m/s}^2}}$$

$= 1.86$ s.

b. How far (horizontally) from the elevator will the pit land?

$v_x = 7.0$ m/s, so

$x = v_x t = (7.0\text{ m/s})(1.86\text{ s})$
$= 13$ m

c. He throws the next pit when the elevator is at the same height but moving upward at a constant 8.5 m/s velocity. How long will it take this pit to land?

Now $v_y = 8.5$ m/s and

$y = v_y t + \frac{1}{2}gt^2$ becomes

$$-17.0\text{ m} = (8.5\text{ m/s})t + \frac{1}{2}(-9.80\text{ m/s})t^2,$$

or $(4.90\text{ m/s}^2)t^2 - (8.5\text{ m/s})t - 17.0\text{ m} = 0$. Using the quadratic formula to solve for t,

$$t = \frac{8.5\text{m/s} \pm \sqrt{(8.5\text{m/s})^2 - 4(4.90\text{m/s}^2)(-17.0\text{m})}}{2(4.90\text{m/s}^2)}$$

$$= \frac{8.50\text{ m/s} \pm 20.1\text{ m/s}}{9.80\text{ m/s}^2}.$$

Choosing the + sign to make flight time position $t = 2.92$ s.

d. How far away will this pit land?

$v_x = 7.0$ m/s, so
$x = v_x t = (7.0\text{ m/s})(2.92\text{ s})$
$= 20$ m

Practice Problems

page 145

9. a. Suppose the mass of the rubber stopper in the Example Problem on page 145 is doubled, but all other given quantities remain the same. How would the velocity, acceleration, and force change?

Since r and T remain the same,

$v = \frac{2\pi r}{T}$ and $a = \frac{v^2}{r}$ remain the same. The new value of the mass is $m' = 2m$. The new force is $F' = m'a = 2ma = 2F$, double the original force.

b. If the radius in the Example Problem were twice as large, but all other given quantities remained the same, how would velocity, acceleration, and force change?

The new radius is $r' = 2r$, so the new velocity is $v' = \frac{2\pi r'}{T} = \frac{2\pi(2r)}{T} = 2v$, twice the original velocity. The new acceleration is $a' = \frac{(v')^2}{r'} = \frac{(2v)^2}{2r} = 2a$, twice the original. The new force is $F' = ma' = m(2a) = 2F$, twice the original.

c. Finally, if the stopper were swung in the same circle so it had a period half as large as in the example, how would the answers change?

new velocity, $v' = \frac{2\pi r}{T'} = \frac{2\pi r}{\left(\frac{1}{2}T\right)} = 2v$, twice the original;

new acceleration, $a' = \frac{(v')^2}{r} = \frac{(2v)^2}{r} = 4a$, four times original;
new force, $F' = ma' = m(4a) = 4F$, four times original

10. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m.

a. Find the centripetal acceleration of the runner.

$$a_c = \frac{v^2}{r} = \frac{(8.8 \text{ m/s})^2}{25 \text{ m}} = 3.1 \text{ m/s}^2$$

b. What supplies the force needed to give this acceleration to the runner?

The track friction force acting on the runner's shoes

11. Racing on a flat track, a car going 32 m/s rounds a curve 56 m in radius.

a. What is the car's centripetal acceleration?

$$a_c = \frac{v^2}{r} = \frac{(32 \text{ m/s})^2}{56 \text{ m}} = 18 \text{ m/s}^2$$

b. What would be the minimum coefficient of static friction between tires and road that would be needed for the car to round the curve without skidding?

Recall $F_f = \mu F_n$. The friction force must supply the centripetal force so $F_f = ma_c$. The normal force is $F_N = -W = -mg$. The coefficient of friction must be at least

$$\mu = \frac{F_f}{F_N} = \frac{ma_c}{-mg} = \frac{a_c}{-g} = \frac{18 \text{ m/s}^2}{-(-9.80 \text{ m/s}^2)}$$

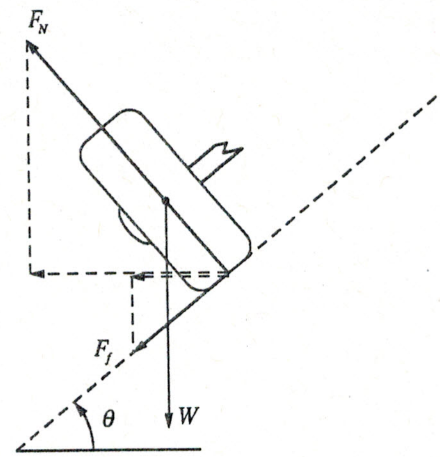
$= 1.8.$

page 146

12. A racing car rounds a curve that is banked.

a. Sketch the auto tire on the incline, drawing vectors representing all the forces on the tire.

Practice Problems



b. Components of what two forces provide the centripetal acceleration for the auto tire, and therefore, the auto?

The horizontal component of the normal force and the horizontal component of the friction force. These component forces, $F_N \sin\theta$ and $F_f \cos\theta$, are shown by dotted lines in the figure.

page 149

13. What is the length of a simple pendulum whose period is 1.00 s? Assume normal g.

From $T = 2\pi\sqrt{\frac{l}{g}}$, we obtain

$$l = \frac{gT^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2)(1.00 \text{ s})^2}{4\pi^2} = 0.248 \text{ m}.$$

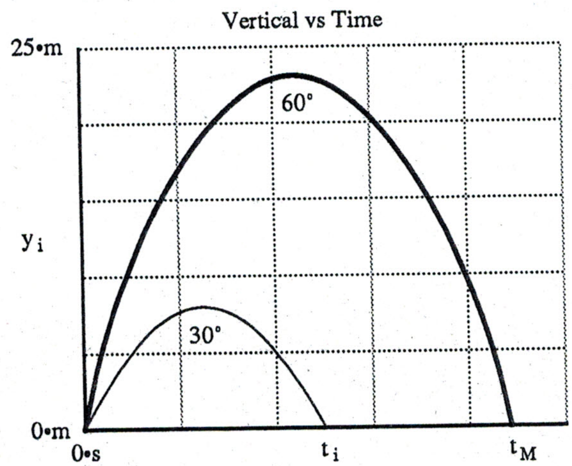
14. A future astronaut lands on a planet with an unknown value of g. She finds that the period of a pendulum 0.65 m long is 2.8 s. What is g for the surface of this planet?

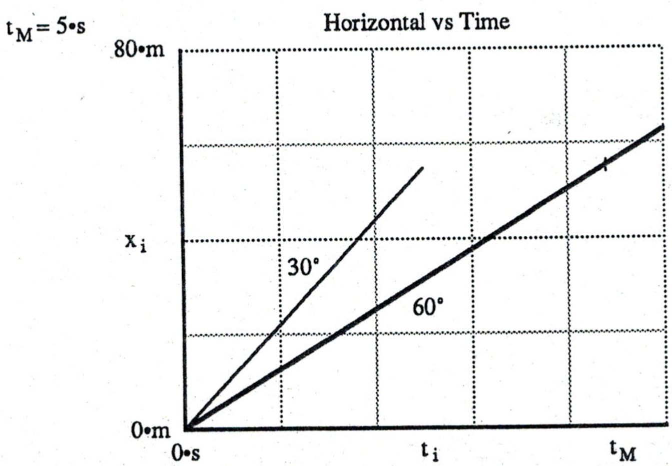
From $T = 2\pi\sqrt{\frac{l}{g}}$, we obtain

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2(0.65 \text{ m})}{(2.85)^2} = 3.3 \text{ m/s}^2.$$

pages 152–153

1. Assuming that the two baseballs in Figure 7–19 have the same velocity, 25 m/s, draw two separate graphs using y as a function of t and x as a function of t for each ball.





Ignore x_i when $y_i < 0$

2. A stone is thrown horizontally at 8.0 m/s from a cliff 78.4 m high. How far from the base of the cliff does the stone strike the ground?

$$y = v_y t. + \frac{1}{2}gt^2$$

Since initial vertical velocity is zero,

$$t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{(2)(-78.4\text{ m})}{-9.8\text{ m/s}^2}} = 4.0\text{ s}$$

$$d = \bar{v}t = (8.0\text{ m/s})(4.0\text{ s}) = 32\text{ m}$$

Chapter Review Problems

3. A toy car runs off the edge of a table that is 1.225 m high. If the car lands 0.400 m from the base of the table,

a. how long does it take for the car to fall to the floor?

$y = v_y t + \frac{1}{2}gt^2$, since initial velocity is zero,

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(-1.225\text{ m})}{(-9.80\text{ m/s}^2)}} = 0.500\text{ s}$$

b. what is the horizontal velocity of the car?

$$v_x = x/t = \frac{0.40\text{ m}}{0.50\text{ s}} = 0.800\text{ m/s}$$

4. Janet jumps off a high diving platform with a horizontal velocity of 2.8 m/s and lands in the water 2.6 s later. How high is the platform, and how far from the base of the platform does she land?

$$y = v_y t = \frac{1}{2}gt^2$$

$$= 0(2.6\text{ m/s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(2.6\text{ s})^2$$

$= -33$ m, so the platform is 33 m high.

$$x = v_x t = (2.8\text{ m/s})(2.6\text{ s}) = 7.3\text{ m}$$

5. An airplane traveling 1001 m above the ocean at 125 km/h is to drop a box of supplies to shipwrecked victims below.

a. How many seconds before being directly overhead should the box be dropped?

$$y = v_y t + \frac{1}{2}gt$$

$$-1001\text{ m} = 0(t) + \frac{1}{2}(-9.8\text{ m/s}^2)t^2,$$

$$t = 14.3\text{ s}$$

b. What is the horizontal distance between the plane and victims when the box is dropped?

$$y_x t = 125\text{ km/h}\left[\frac{1\text{ h}}{3600\text{ s}}\right]\left[\frac{1000\text{ m}}{1\text{ km}}\right]$$

$$= 34.7\text{ m/s}$$

$$x = v_x t = (34.7\text{ m/s})(14.3\text{ s}) = 496\text{ m}$$

6. Divers at Acapulco dive from a cliff that is 61 m high. If the rocks below the cliff extend outward for 23 m, what is the minimum horizontal velocity a diver must have to clear the rocks safely?

$y = v_y t + \frac{1}{2}gt^2$, since intial velocity is zero,

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(-61\text{ m})}{(-9.8\text{ m/s}^2)}} = \sqrt{12.4\text{ s}^2} = 3.5\text{ s}$$

$$v_x = x/t = \frac{23\text{ m}}{3.5\text{ s}} = 6.6\text{ m/s}$$

7. A dart player throws a dart horizontally at a speed of +12.4 m/s. The dart hits the board 0.32 m below the height from which it was thrown. How far away is the player from the board?

$y = v_y t + \frac{1}{2}gt^2$, since $v_y = 0$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(-0.32\text{ m})}{(-9.80\text{ m/s}^2)}} = 0.26\text{ s}$$

Now $x = v_x t = (12.4\text{ m/s})(0.26\text{ s}) = 3.2\text{ m}$

8. An arrow is shot at a 30.0° angle with the horizontal. It has a velocity of 49 m/s.

The components of the initial velocity are
$v_x = v_i \cos\theta = (49\text{ m/s}) \cos 30.0 = 42\text{ m/s}$
$v_y = v_i \sin\theta = (49\text{ m/s}) \sin 30.0 = 25\text{ m/s}$

a. How high will the arrow go?

At the high point $v_y = 0$, so
$v_{yf}^2 = v_{yi}^2 + 2gd$ therefore

$$d = \frac{-(v_{yi})^2}{2g} = \frac{-(25\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 32\text{ m}$$

b. What horizontal distance will the arrow travel?

$y = v_y t + \frac{1}{2}gt^2$,

but the arrow lands at the same height, so

$y = 0$ and $0 = (v_y + \frac{1}{2}gt)t$ so $t = 0$ or

$$t = \frac{-v_y}{\frac{1}{2}(g)} = \frac{-25\text{ m/s}}{\frac{1}{2}(-9.80\text{ m/s}^2)} = 5.1\text{ s}$$

and $x = v_x t = (42\text{ m/s})(5.1\text{ s})$
$= 2.1 \times 10^2\text{ m}$

Chapter Review Problems

9. A pitched ball is hit by a batter at a 45° angle. It just clears the outfield fence, 98 m away. Find the velocity of the ball when it left the bat. Assume the fence is the same height as the pitch.

The components of the initial velocity are
$v_x = v_i \cos\theta$ and $v_y = v_i \sin\theta$

Now $x = v_x t = (v_i \cos\theta)t$, so $t = \dfrac{x}{v_i \cos\theta}$

And $y = v_y t + \frac{1}{2}gt^2$, but $y = 0$, so

$$0 = \left[v_y + \frac{1}{2}gt\right]t \text{ so } t = 0 \text{ or } v_y + \frac{1}{2}gt = 0,$$

from above $v_i \sin\theta + \frac{1}{2}g\left[\dfrac{x}{v_i \cos\theta}\right] = 0$

Multiply by $v_i \cos\theta$ gives

$v_i^2 \sin\theta \cos\theta + \frac{1}{2}gx = 0$, so

$$v_i^2 = \frac{-gx}{2\sin\theta\cos\theta} = \frac{-(-9.80\text{ m/s}^2)(98\text{ m})}{2(\sin 45)(\cos 45)}$$

$= 9.6 \times 10^2\text{ m}^2/\text{s}^2$

$v_i = 31$ m/s at 45°

10. Trailing by two points, and with only 2.0 s remaining in a basketball game, a player makes a jump–shot at an angle of 60° with the horizontal, giving the ball a velocity of 10 m/s. The ball is released at the height of the basket, 3.05 m above the floor. Yes! It's a score!

The components of the initial velocity are
$v_x = v_i \cos\theta = (10\text{ m/s})(\cos 60) = 5.0\text{ m/s}$
$v_y = v_i \sin\theta = (10\text{ m/s})(\sin 60) = 8.7\text{ m/s}$

a. How much time is left in the game when the basket is made?

$y = v_y t + \frac{1}{2}gt^2$, but $y = 0$, so

$0 = (v_y + \frac{1}{2}gt)t$ and $t = 0$ or

$$t = \frac{-v_y}{\frac{1}{2}g} = \frac{-8.7\text{ m/s}}{\frac{1}{2}(-9.80\text{ m/s}^2)} = 1.8\text{ s}$$

Time left = 2.0 s − 1.8 s = 0.2 s

7

Chapter Review Problems

b. Shots made outside a semicircle of 6.02-m radius from a spot directly beneath the basket are awarded 3 points, while those inside score 2 points. Did the player tie the game or put the team ahead?

$x = v_x t = 5.0 \text{ m/s}(1.8 \text{ s}) = 9.0 \text{ m}$
She got 3 points to win the game.

11. A basketball player tries to make a half-court jump-shot, releasing the ball at the height of the basket. Assuming the ball is launched at 51.0°, 14.0 m from the basket, what velocity must the player give the ball?

The components of the initial velocity are
$v_x = v_i \cos\theta$ and $v_y = v_i \sin\theta$

Now $x = v_x t = (v_i \cos\theta)t$, so $t = \dfrac{x}{v_i \cos\theta}$

And $y = v_y t + \frac{1}{2}gt^2$, but $y = 0$,

so $0 = \left[v_y + \frac{1}{2}gt\right]t$.

Therefore, $t = 0$ or $v_y + \frac{1}{2}gt = 0$ from above

$$v_i \sin\theta + \frac{1}{2}g\left[\frac{x}{v_i \cos\theta}\right] = 0$$

multiply by $v_i \cos\theta$ gives

$$v_i^2 \sin\theta \cos\theta + \frac{1}{2}gx = 0, \text{ so}$$

$$v_i = \sqrt{\frac{-gx}{2 \sin\theta \cos\theta}}$$

$$= \sqrt{\frac{-(-9.80 \text{ m/s}^2)(14.0 \text{ m})}{2(\sin 51)(\cos 51)}}$$

$= 11.8$ m/s, 51.0°

12. A baseball is hit at 30.0 m/s at an angle of 53.0° with the horizontal. Immediately an outfielder runs 4.00 m/s toward the infield and catches the ball bat the same height it was hit. What was the original distance between the batter and the outfielder?

The components of the initial velocity are
$v_x = v_i \cos\theta = 30.0 \cos 53.0° = 18.1 \text{ m/s}$
$v_y = v_i \sin\theta = 30.0 \sin 53.0° = 24.0 \text{ m/s}$

$y = v_y t + \frac{1}{2}gt^2$, but $y = 0$, so

$0 = \left[v_y + \frac{1}{2}gt\right]t$ therefore $t = 0$ or

$$t = \frac{-v_y}{\frac{1}{2}g} = \frac{-24.0 \text{ m/s}}{\frac{1}{2}(-9.80 \text{ m/s}^2)} = 4.90 \text{ s}$$

The horizontal distance the ball travels is
$x = v_x t = (18.1 \text{ m/s})(4.90 \text{ s}) = 88.5 \text{ m}$
The distance the outfielder travels is
$x = v_x t = (4.00 \text{ m/s})(4.90 \text{ s}) = 19.6 \text{ m}$
original separation $= 88.5 \text{ m} + 19.6 \text{ m}$
$= 108.3 \text{ m}$

13. It takes a 615-kg racing car 14.3 s to travel at a uniform speed around a circular racetrack of 50.0 m radius.

a. What is the acceleration of the car?

$v = 2\pi r/T = 2\pi(50.0 \text{ m})/(14.3 \text{ s})$
$= 22.0 \text{ m/s}$ $a_c = v^2/r$
$= (22.0 \text{ m/s})^2/(50.0 \text{ m}) = 9.65 \text{ m/s}^2$

b. What average force must the track exert on the car's tires to produce this acceleration?

$F_c = ma_c = (615 \text{ kg})(9.65 \text{ m/s}^2)$
$= 5.94 \times 10^3 \text{ N}$

14. An athlete whirls a 7.00-kg hammer tied to the end of a 1.3-m chain in a horizontal circle. The hammer moves at the rate of 1.0 rev/s.

a. What is the centripetal acceleration of the hammer?

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{(4\pi^2)(1.3 \text{ m})}{(1.0 \text{ s})^2} = 51 \text{ m/s}^2$$

Chapter Review Problems

b. What is the tension in the chain?

$F_c = ma_c = (7.00 \text{ kg})(51 \text{ m/s}^2)$
$= 3.6 \times 10^2 \text{ N}$

15. Sue whirls a yo–yo in a horizontal circle. The yo–yo has a mass of 0.20 kg and is attached to a string 0.80 m long.

a. If the yo–yo makes 1.0 complete revolution each second, what force does the string exert on it?

$f = 1.0 \text{ Hz, so } T = \frac{1}{f} = 1.0 \text{ s}$

$$F_c = m\frac{4\pi^2 r}{T^2} = \frac{(0.20 \text{ kg})(4\pi^2)(0.80 \text{ m})}{(1.0 \text{ s})^2}$$

$= 6.3 \text{ N}$

b. If Sue increases the speed of the yo–yo to 2.0 revolutions per second, what force does the string now exert?

$f = 2.0 \text{ Hz, so } T = \frac{1}{f} = 0.50 \text{ s}$

$$F_c = m\frac{4\pi^2 r}{T^2} = \frac{(0.20 \text{ kg})(4\pi^2)(0.80 \text{ m})}{(0.50 \text{ s})^2}$$

$= 25 \text{ N}$

c. What is the ratio of answer (b) to (a)? Why?

25 N:6.3 N is 4:1 because velocity has doubled, acceleration doubled, squared; or multiplied by four.

16. A coin is placed on a stereo record revolving at 33 1/3 revolutions per minute.

a. In what direction is the acceleration of the coin, if any?

The acceleration is toward the center of the record.

b. Find the acceleration of the coin when it is placed 5, 10, and 15 cm from the center of the record.

$$T = \frac{1}{f} = \frac{1}{33\frac{1}{3}} = (0.0300)\left[\frac{60 \text{ s}}{1 \text{ min}}\right] = 1.80 \text{ s}$$

$$\text{for } r = 5.0;\ a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(0.05 \text{ m})}{(1.80 \text{ s})^2}$$

$= 0.61 \text{ m/s}^2$

$$\text{for } r = 10;\ ac = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(0.10 \text{ m})}{(1.80 \text{ s})^2}$$

$= 1.2 \text{ m/s}^2$

$$\text{for } r = 15;\ a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(0.15 \text{ m})}{(1.80 \text{ s})^2}$$

$= 1.8 \text{ m/s}^2$

c. What force accelerates the coin?

Frictional force between coin and record.

d. At which of the three radii listed in b would the coin be most likely to fly off? Why?

15 cm
The largest radius because force to hold it is the greatest.

17. According to the *Guinness Book of World Records*, (1990 edition, p 169) the highest rotary speed ever attained was 2010 m/s (4500 mph). The rotating rod was 15.3 cm (6 in.) long. Assume the speed quoted is that of the end of the rod.

a. What is the centripetal acceleration of the end of the rod?

$$a_c = \frac{v^2}{r} = \frac{(2010 \text{ m/s})^2}{0.153 \text{ m}} = 2.64 \times 10^7 \text{ m/s}^2$$

b. If you were to attach a 1.00–g object to the end of the rod, what force would be needed to hold it on the rod?

$F_c = ma_c = (0.00100 \text{ kg})(2.64 \times 10^7 \text{ m/s}^2)$
$= 2.64 \times 10^4 \text{ N}$

7

c. What is the period of rotation of the rod?

$a_c = \frac{4\pi^2 r}{T^2}$, so

$$T = \sqrt{\frac{4\pi^2 r}{a_c}} = 2\pi\sqrt{\frac{r}{a_c}} = 2\pi\sqrt{\frac{0.153\ \text{m}}{2.64 \times 10^7\ \text{m/s}^2}}$$

$= 4.78 \times 10^{-4}$ s

18. Refer to Figure 7–8. The carnival ride has a 2.0–m radius and rotates 1.1 times per second.

$$T = \frac{1}{f} = \frac{1}{1.1} = 0.91\ \text{s}$$

a. Find the speed of a rider.

$$v = \frac{\Delta d}{\Delta t} = \frac{2\pi r}{T} = \frac{2\pi(2.0\ \text{m})}{(0.91\ \text{s})} = 14\ \text{m/s}$$

b. Find the centripetal acceleration of a rider.

$$a_c = \frac{v^2}{r} = \frac{(14\ \text{m/s})^2}{2.0\ \text{m}} = 98\ \text{m/s}^2$$

c. What produces this acceleration?

Force of the drum walls.

d. When the floor drops down, riders are held up by a friction. What coefficient of friction is needed to keep the riders from slipping?

Downward force of gravity $F = mg$.
Frictional force $F_f = \mu F_N$.
F_N is the force of the drum, ma_c.
To balance $g = \mu a_c$, so we need
$\mu = g/a_c = 0.10$.

19. An early major objection to the idea that Earth is spinning on its axis was that Earth would turn so fast at the equator that people would be thrown off into space. Show the error in this logic by calculating

a. the speed of a 97–kg person at the equator. The radius of Earth is about 6400 km.

$$v = \frac{\Delta d}{\Delta t} = \frac{2\pi r}{T} = \frac{2\pi(6.40 \times 10^6\ \text{m})}{\left[\frac{24\ \text{h}}{1}\right]\left[\frac{3600\ \text{s}}{1\ \text{h}}\right]}$$

$= 465$ m/s

Chapter Review Problems

b. the centripetal force on the person.

$$F_c = ma_c = \frac{mv^2}{r} = \frac{(97\ \text{kg})(465\ \text{m/s})^2}{(6.40 \times 10\ \text{m})}$$

$= 3.3$ N

c. the weight of the person.

$F = mg = (97\ \text{kg})(9.80\ \text{m/s}^2) = 950$ N
$= 9.5 \times 10^2$ N

20. Friction provides the centripetal force necessary for a car to travel around a flat circular race track. What is the maximum speed at which a car can safely travel around a circular track of radius 80.0 m if the coefficient of friction between tire and road is 0.30?

$$F_c = F_f = \mu F_N = \mu mg$$

But $F_c = \frac{mv^2}{r}$, thus $\frac{mv^2}{r} = \mu mg$ and the mass of the car divides out to give $r^2 = \mu gr$, so

$$v = \sqrt{\mu gr} = \sqrt{(0.30)(9.80\ \text{m/s}^2)(80.0\ \text{m})} = 15\ \text{m/s}$$

21. A pendulum has a length of 0.67 m.

a. Find its period.

$$T = 2\pi\sqrt{(\ell/g)} = 1.6\ \text{s.}$$

b. How long would the pendulum have to be to double the period?

Since the period is proportional to the square root of the length, the pendulum would have to be four times as long, or 2.7 m.

c. Why is your answer to part b not just double the length?

The period is proportional to the *square root* of the length, not the length, so the answer cannot be doubled.

22. Find the length of a pendulum oscillating on the moon that would have the same period as a 1.0–m pendulum oscillating on Earth. The moon's gravity is one–sixth of Earth's gravity.

$$T = 2\pi\sqrt{\frac{\ell_e}{g_e}} = 2\pi\sqrt{\frac{\ell_m}{g_m}},\ \sqrt{\frac{\ell_e}{g_e}} = \sqrt{\frac{\ell_m}{g_m}} \text{ so}$$

$$\frac{\ell_e}{g_e} = \frac{\ell_m}{g_m},\ \frac{1.0\text{ m}}{9.8\text{ m/s}^2} = \frac{\ell_m}{1.6\text{ m/s}^2} = 0.16\text{ m}$$

Supplemental Problems (Appendix B)

1. A ball falls from rest from a height of 490 m.

a. How long does it remain in the air?

$y = v_y t + \frac{1}{2}gt^2$, since initial vertical velocity is zero,

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(-490\text{ m})}{(-9.80\text{ m/s}^2)}} = 10.0\text{ s}$$

b. If the ball has a horizontal velocity of 2.00×10^2 m/s when it begins its fall, what horizontal displacement will it have?

$$x = v_x t = (2.00 \times 10^2\text{ m/s})(1.00 \times 10^1\text{ s}) = 2.00 \times 10^3\text{ m}$$

2. An archer stands 40.0 m from the target. If the arrow is shot horizontally with a velocity of 90.0 m/s, how far above the bull's–eye must he aim to compensate for gravity pulling his arrow downward?

$$t = \frac{x}{v_x} = \frac{40.0\text{ m}}{90.0\text{ m/s}} = 0.444\text{ s}$$

$$y = v_y t + \frac{1}{2}gt^2$$

$$= 0(0.444\text{ s}) + \frac{1}{2}(9.80\text{ m/s}^2)(0.444\text{ s})^2$$

$$= 0.966\text{ m}$$

3. A bridge is 176.4 m above a river. If a lead–weighted fishing line is thrown from the bridge with a horizontal velocity of 22.0 m/s, how far has it moved horizontally when it hits the water?

$$y = v_y t + \frac{1}{2}gt^2$$

Since initial velocity is zero,

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(-176.4\text{ m})}{(-9.80\text{ m/s}^2)}} = 6.00\text{ s}$$

$$x = v_x t = (22.0\text{ m/s})(6.00\text{ s}) = 132\text{ m}$$

4. A beach ball, moving with a speed of +1.27 m/s, rolls off a pier and hits the water 0.75 m from the end of the pier. How high is the pier above the water?

We need to know how long it takes the ball to hit the water in order to use $y = v_y t + \frac{1}{2}gt^2$, $v_y = 0$, to calculate pier height. This time is determined by the horizontal motion $x = v_x t$.
$t = x/v_x = (0.75\text{ m})/(1.27\text{ m/s}) = 0.59$ s.
This gives a vertical displacement

$$y = \frac{1}{2}gt^2 = \left[\frac{1}{2}\right](-9.80\text{ m/s}^2)(0.59\text{ s})^2 = -1.7\text{ m}$$

and hence a pier height of 1.7 m.

5. Pete has a tendency to drop his bowling ball on his release. Instead of having the ball on the floor at the completion of his swing, Pete lets go with the ball 0.35 m above the floor. If he throws it horizontally with a velocity of 6.3 m/s, what distance does it travel before you hear a "thud?"

$y = v_y t + \frac{1}{2}gt^2$ where $v_y = 0$. Taking downward position, the time to hit floor is

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(0.35\text{ m})}{9.8\text{ m/s}^2}} = 0.275 \text{ so travel}$$

distance is $x = v_x t = (6.3\text{ m/s})(0.275) = 1.7$ m

7

Supplemental Problems

6. A discus is released at an angle of 45° and a velocity of 24.0 m/s.

a. How long does it stay in the air?

$v_y = v_i \sin 45° = (24.0\text{ m/s}) \sin 45°$
$= 17.0\text{ m/s}$
so time to maximum height is

$$t = \frac{v_{top} - v_y}{a} = \frac{0 - 17.0\text{ m/s}}{-9.80\text{ m/s}^2} = 1.73\text{ s}$$ and

by symmetry total time is 3.46 s.

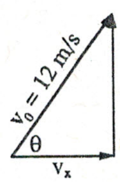
b. What horizontal distance does it travel?

$v_x = v_i \cos 45° = (24.0\text{ m/s}) \cos 45°$
$= 17.0\text{ m/s}$
so $x = v_x t = 17.0\text{ m/s}(3.46\text{ s}) = 58.8\text{ m}$

7. A shot put is released with a velocity of 12 m/s and stays in the air for 2.0 s.

a. At what angle with the horizontal was it released?

$v_f = v_i + at$ where $v_f = 0$ at maximum height and $v_i = v_y$.
Since time to maximum height is 1.0 s,
$v_y = v_f - at = 0 - (-9.8\text{ m/s}^2)(1.0\text{ s})$
$= 9.8\text{ m/s}$ where upward is taken positive.



$$\sin\theta = \frac{v_y}{v_o} = \frac{9.8\text{ m/s}}{12\text{ m/s}} = 0.817$$
$\theta = 55°$

b. What horizontal distance did it travel?

$v_x = v_o \cos 55° = (12\text{ m/s}) \cos 55°$
$= 6.9\text{ m/s}$
so $x = v_x t = (6.9\text{ m/s})(2.0\text{ s}) = 14\text{ m}$

8. A football is kicked at 45° and travels 82 m before hitting the ground.

a. What was its initial velocity?

$x = v_x t$ and $y = v_y t + \frac{1}{2}gt^2$. At end $y = 0$,

so $0 = v_y t + \frac{1}{2}gt^2 = t(vy + gt/2)$ and

$t = -2v_y/g$. But $t = x/v_x$,

so $x/v_x = -2v_y/g$ or $v_x v_y = -\frac{1}{2}xg$.

Now $v_x = v_o \cos\theta$ and $v_y = v_o \sin\theta$,
so $v_x v_y = v_o^2 \cos\theta \sin\theta$, or

$v_o^2 = -\frac{1}{2}xg/\cos\theta \sin\theta$.

Here $v_o^2 = \dfrac{-1/2(82\text{ m})(-9.80\text{ m/s}^2)}{(0.707)(0.707)}$

$= 804\text{ m}^2/\text{s}^2$
or $v_o = 28\text{ m/s}$.

b. How long was it in the air?

$v_x = v_o \cos 45° = (28\text{ m/s}) \cos 45°$
$= 20\text{ m/s}$
so $t = x/v_x = (82\text{ m})/(20\text{ m/s}) = 4.1\text{ s}$.

c. How high did it go?

Max height occurs at half the flight time, so since
$v_y = v_o \sin 45° = (28\text{ m/s}) \sin 45°$
$= 20\text{ m/s}$,

$y = (20\text{ m/s})(2.1\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(2.1\text{ s})^2$

$= 20\text{ m}$

9. A golf ball is hit with a velocity of 24.5 m/s at 35.0° above the horizontal. Find

a. the range of the ball.

$v_x = v_i \cos\theta = (24.5 \text{ m/s})(0.819)$
$= 20.1 \text{ m/s}$
$v_y = v_i \sin\theta = (24.5 \text{ m/s})(0.574)$
$= 14.1 \text{ m/s.}$

$$y = v_y t + \frac{1}{2}gt^2 = t(v_y + gt/2)$$

When $y = 0$,
$t = -2v_y/g = -2(14.1 \text{ m/s})/(-9.80 \text{ m/s}^2)$
$= 2.88 \text{ s, so } x = v_x t = 57.9 \text{ m}$

b. the maximum height of the ball.

In half the flight time, $\left[\frac{1}{2}\right](2.88 \text{ s})$, it falls

$$y = \left[\frac{1}{2}\right]gt^2 = \left[\frac{1}{2}\right](-9.80 \text{ m/s}^2)(1.44 \text{ s})^2$$

$= -10.2$ m,
so its maximum height is 10.2 m.

10. A carnival clown rides a motorcycle down a ramp and around a "loop-the-loop." If the loop has a radius of 18 m, what is the slowest speed the rider can have at the top of the loop to avoid falling? **Hint:** At this slowest speed, at the top of the loop, the clown's weight is equal to the centripetal force.

$F = ma$ and $F_c = W$, so
$mv^2/r = mg$;
$v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(18 \text{ m})} = 13 \text{ m/s}$

11. A 75-kg pilot flies a plane in a loop. At the top of the loop, where the plane is completely upside-down for an instant, the pilot hangs freely in the seat and does not push against the seat belt. The airspeed indicator reads 120 m/s. What is the radius of the plane's loop?

Since the centripetal force is exactly equal to the weight of the pilot, $\frac{mv^2}{r} = mg$, or

$$r = \frac{v^2}{g} = \frac{(120 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 1.5 \times 10^3 \text{ m}$$

Supplemental Problems

12. A 2.0-kg object is attached to a 1.5 m long string and swung in a vertical circle at a constant speed of 12 m/s.

a. What is the tension in the string when the object is at the bottom of its path?

$F_c = mv^2/r = (2.0 \text{ kg})(12 \text{ m/s}^2)/(1.5 \text{ m})$
$= 1.9 \times 10^2 \text{ N}$

$W = mg = (2.0 \text{ kg})(9.8 \text{ m/s}^2) = 20 \text{ N}$
$T = F_c + W$
$= 1.9 \times 10^2 \text{ N} + 0.20 \times 10^2 \text{ N}$
$= 2.1 \times 10^2 \text{ N}$

b. What is the tension in the string when the object is at the top of its path?

$T = F_c - W = 1.9 \times 10^2 - 0.20 \times 10^2 \text{ N}$
$= 1.7 \times 10^2 \text{ N}$

13. A 60.0-kg speed skater with a velocity of 18.0 m/s comes into a curve of 20.0-m radius. How much friction must be exerted between the skates and the ice to negotiate the curve?

$$F_f = F_c = \frac{mv^2}{r} = \frac{(60.0 \text{ kg})(18.0 \text{ m/s})^2}{20.0 \text{ m}} = 972 \text{ N}$$

14. A 20.0-kg child wishes to balance on a seesaw with a child of 32.0 kg. If the smaller child sits 3.2 m from the pivot, where must the larger child sit?

$m_1 g d_1 = m_2 g d_2$
Since g is common to both sides,
$(20.0)(3.2) = (32.0)(d_2)$ and $d_2 = 2.0$ m

15. A pendulum has a length of 1.00 m.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

a. What is its period on Earth?

$$T = 2\pi\sqrt{\frac{1.00 \text{ m}}{9.80 \text{ m/s}^2}} = 2.01 \text{ s}$$

b. What is its period on the moon where the acceleration due to gravity is 1.67 m/s²?

$$T = 2\pi\sqrt{\frac{1.00 \text{ m}}{1.67 \text{ m/s}^2}} = 4.86 \text{ s}$$

7

16. The period of an object oscillating on a spring is

$$T = 2\pi\sqrt{\frac{m}{k}},$$

where m is the mass of the object and k is the spring constant which indicates the force necessary to produce a unit elongation of the spring. The period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

a. What mass will produce a 1.0-s period of oscillation if it is attached to a spring with a spring constant of 4.0 N/m?

$T = 2\pi\sqrt{m/k}$, so
$m = kT^2/4\pi^2 = (4.0\ \text{N/m})(1.0\ \text{s})^2/(4)(\pi^2)$
$= 0.10\ \text{kg}$

b. What length pendulum will produce a period of 1.0 s?

$T = 2\pi\sqrt{l/g}$, so $l = gT^2/4\pi^2$
$l = (9.80\ \text{m/s}^2)(1.0\ \text{s})^2/(4)(\pi^2)$
$= 0.25\ \text{m}$

c. How would the harmonic oscillator and the pendulum have to be modified in order to produce 1.0-s periods on the surface of the moon, where g is 1.6 m/s^2?

No change is necessary for the harmonic oscillator. For the pendulum, since l is proportional to g,
$l' = g'l/g = (1.6\ \text{m/s}^2)(0.25\ \text{m})/(9.8\ \text{m/s}^2)$
$= 0.041\ \text{m}.$
The pendulum must be shortened to 4.1 cm.