# Chapter 6: Vectors

#### **Practice Problems**

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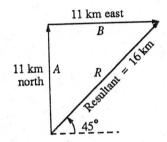
Draw vector diagrams to solve each problem.

- After walking 11 km due north from camp, a hiker then walks 11 km due east.
  - a. What is the total distance walked by the hiker?

$$11 \text{ km} + 11 \text{ km} = 22 \text{ km}$$

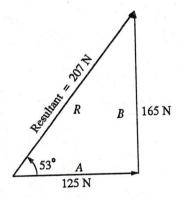
b. Determine the total displacement from the starting point.

16 km



2. Two boys push on a box. One pushes with a force of 125 N to the east. The other exerts a force of 165 N to the north. What is the size and direction of the resultant force on the box?

207 N, 53°, north of east

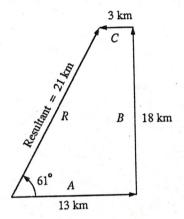


### Practice Problems

- An explorer walks 13 km due east, then 18 km north, and finally 3 km west.
  - a. What is the total distance walked?

$$13 \text{ km} + 18 \text{ km} + 3 \text{ km} = 34 \text{ km}$$

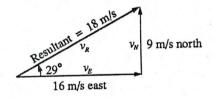
- b. What is the resulting displacement of the explorer from the starting point?
  - 21 km, 61°, north of east



Draw vector diagrams to solve each problem.

- A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.
  - a. What is the resultant velocity (speed and direction) of the boat?

18 m/s, 29°, north of east



b. If the river is 136 m wide, how long does it take the motorboat to reach the other side?

$$t = \frac{d}{v_{\rm E}} = \frac{136 \text{ m}}{16 \text{ m/s}} = 8.5 \text{ s}$$

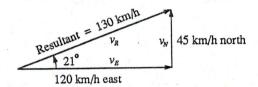
### Practice Problems

c. How far downstream is the boat when it reaches the other side of the river?

$$d = v_N t = (9.0 \text{ m/s})(8.5 \text{ s}) = 77 \text{ m}$$

5. While flying due east at 120 km/h, an airplane is also carried northward at 45 km/h by the wind blowing due north. What is the plane's resultant velocity?

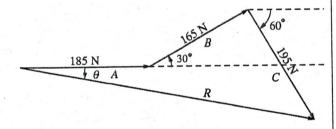
130 km/h, 21°, north of east



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6. Three teenagers push a heavy crate across the floor. Dion pushes with a force of 185 N at 0°. Shirley exerts a force of 165 N at 30°, while Joan pushes with a 195 N force at 300°. What is the resultant force on the crate?

434 N, 11.5°, south of east



#### page 115

7. A 110-N force and a 55-N force both act on an object point P. The 110-N force acts at 90°. The 55-N force acts at 0°. What is the magnitude and direction of the resultant force?

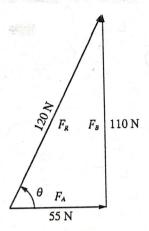
$$F_{R}^{2} = F_{A}^{2} + F_{B}^{2}$$

$$F_{R} = \sqrt{(55 \text{ N})^{2} + (110 \text{ N})^{2}} = 120 \text{ N}$$

$$\tan \theta = \frac{110}{55} = 2.0$$

$$\theta = 63^{\circ}$$

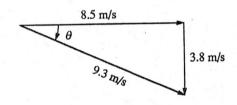
#### Practice Problems



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- A motorboat travels at 8.5 m/s. It heads straight across a river 110 m wide.
  - a. If the water flows downstream at a rate of 3.8 m/s, what is the boat's resultant velocity?

$$v_{\rm R} = \sqrt{(8.5 \text{ m/s})^2 + (3.8 \text{ m/s})^2} = 9.3 \text{ m/s}$$
  
 $\tan \theta = \frac{3.8}{0.5} = 0.45, \ \theta = 24^{\circ}$   
 $v_{\rm R} = 9.3 \text{ m/s} \text{ at } 24^{\circ}$ 



**b.** How much time does it take the boat to reach the opposite shore?

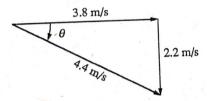
$$t = \frac{d}{v} = \frac{110 \text{ m}}{8.5 \text{ m/s}} = 13 \text{ s}$$

#### Practice Problems

- 9. A boat heads directly across a river 41 m wide at 3.8 m/s. The current is flowing downstream at 2.2 m/s.
  - a. What is the resultant velocity of the boat?

$$v_{\rm R} = \sqrt{(3.8 \text{ m/s})^2 + (2.2 \text{ m/s})^2} = 4.4 \text{ m/s}$$
  
 $\tan \theta = \frac{2.2}{3.8} = 0.58, \ \theta = 30^{\circ}$ 

$$v_{\rm R} = 4.4 \text{ m/s at } 30^{\circ}$$



b. How much time does it take the boat to cross the river?

$$t = \frac{d}{v} = \frac{41 \text{ m}}{3.8 \text{ m/s}} = 11 \text{ s}$$

c. How far downstream is the boat when it reaches the other side?

$$d = vt = (2.2 \text{ m/s})(11 \text{ s}) = 24 \text{ m}$$

10. A 42-km/h wind blows toward 215°, while a plane heads toward 125° at 152 km/h. What is the resultant velocity of the plane?

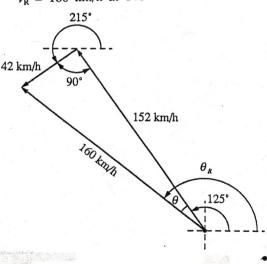
$$v_{R} = \sqrt{(152 \text{ km/h})^{2} + (42 \text{ km/h})^{2}} = 160 \text{ km/h}$$

$$\tan \theta = \frac{42}{152} = 0.276$$

$$\theta = 15^{\circ}$$

$$\theta_{R} = 125^{\circ} + 15^{\circ} = 140^{\circ}$$

$$v_{\rm R} = 160 \text{ km/h} \text{ at } 140^{\circ}$$



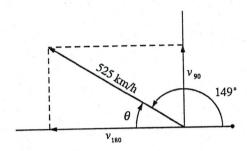
#### Practice Problems

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11. A heavy box is pulled across a wooden floor with a rope. The rope makes an angle of 60° with the floor. A force of 75 N is exerted on the rope. What is the component of the force parallel to the floor?

$$F_h = (75 \text{ N})\cos 60^\circ = 38 \text{ N}$$

12. An airplane flies toward 149° at 525 km/h.



$$\theta = 180^{\circ} - 149^{\circ} = 31^{\circ}$$

What is the component of the plane's velocity

a. toward 90°?

$$v_{90} = v \sin \theta = (525 \text{ km/h}) \sin 31^{\circ}$$
  
= 270 km/h

**b.** toward 180°?

$$v_{180} = v \cos \theta = (525 \text{ km/h}) \cos 31^{\circ}$$
  
= 450 km/h

13. A student exerts a force of 72 N along the handle of a lawn mower to push it across the lawn. Find the horizontal component of this force when the handle is held at an angle with the lawn of

**a.** 60.0°.

$$F_{\rm h} = (72 \text{ N}) \cos 60^{\circ} = 36 \text{ N}$$

**b.** 40.0°.

$$F_{\rm h} = (72 \text{ N}) \cos 40^{\circ} = 55 \text{ N}$$

c. 30.0 °.

$$F_h = (72 \text{ N}) \cos 30^\circ = 62 \text{ N}$$

### (0)

#### Practice Problems

14. A hiker walks 14.7 km at an angle of 305° from east. Find the east—west and north—south components of this walk.

$$\theta = 360^{\circ} - 305^{\circ} = 55^{\circ}$$
 south of east  
east – west component = (14.7 km) cos 55°  
= 8.43 km, east  
north – south component = (14.7 km) sin 55°  
= 12.0 km, south.

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15. Find the resultant force on the log in the last Example Problem if  $F_1$  remains the same and  $F_2$  is changed to 14.0 N at 310.0°.

$$F_{1x} = (12.0 \text{ N}) \cos 10.0^{\circ} = 11.8 \text{ N}$$

$$F_{1y} = (12.0 \text{ N}) \sin 10.0^{\circ} = 2.0 \text{ N}$$

$$F_{2x} = (14.0 \text{ N}) \cos 310.0^{\circ} = 9.0 \text{ N}$$

$$F_{2y} = (14.0 \text{ N}) \sin 310.0^{\circ} = -10.7 \text{ N}$$
where  $\cos 310.0^{\circ} = \cos (-50.0^{\circ}) = \cos 50.0^{\circ}$ 
 $\sin 310.0^{\circ} = \sin (-50.0^{\circ}) = -\sin 50.0^{\circ}$ 

$$F_{\text{net}} = \sqrt{(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2}$$

$$= \sqrt{(11.8 \text{ N} + 9.0 \text{ N})^2 + (2.0 \text{ N} - 10.7 \text{ N})^2}$$

$$= \sqrt{(20.8 \text{ N})^2 + (-8.7 \text{ N})^2} = 22.5 \text{ N}$$

$$\tan \theta = \frac{F_y}{F_x} = \frac{-8.7 \text{ N}}{20.8 \text{ N}} = -0.418$$

$$\theta = -22.7^{\circ} \text{ or } 337.3^{\circ}$$

16. Three people are pulling on a tree. The first person pulls with 15 N at 65.0°; the second with 16 N at 135°; the third with 11 N at 195°. What is the magnitude and direction of the resultant force of the tree?

$$F_{1x} = (15 \text{ N}) \cos 65.0^{\circ} = 6.3 \text{ N}$$

$$F_{1y} = (15 \text{ N}) \sin 65.0^{\circ} = 13.6 \text{ N}$$

$$F_{2x} = (16 \text{ N}) \cos 135^{\circ} = -11.3 \text{ N}$$

$$F_{2y} = (16 \text{ N}) \sin 135^{\circ} = 11.3 \text{ N}$$

$$F_{3x} = (11 \text{ N}) \cos 195^{\circ} = -10.6 \text{ N}$$

$$F_{3y} = (11 \text{ N}) \sin 195^{\circ} = -2.8 \text{ N}$$

$$F_{x} = F_{1x} + F_{2x} + F_{3x}$$

$$= 6.3 \text{ N} + (-11.3 \text{ N}) + (-10.6 \text{ N})$$

$$= -15.6 \text{ N}$$

$$F_{y} = F_{1y} + F_{2y} + F_{3y}$$

$$= 13.6 \text{ N} + 11.3 \text{ N} + (-2.8 \text{ N}) = 22.1 \text{ N}$$

$$F_{\text{net}} = \sqrt{F_{x}^{2} + F_{y}^{2}}$$

$$= \sqrt{(-15.6 \text{ N})^{2} + (22.1 \text{ N})^{2}} = 27.1 \text{ N}$$

$$\tan \theta = \frac{F_{y}}{F_{x}} = \frac{22.1 \text{ N}}{-15.6 \text{ N}} = -1.42$$

Since, from signs of  $F_x$  and  $F_y$ ,  $\theta$  is second quadrant angle  $\theta = 180^{\circ} - 54.8^{\circ} = 125^{\circ}$ 

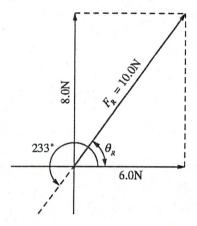
#### Practice Problems

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17. A net force of 55 N acts due west on an object. What added single force on the object produces equilibrium?

55 N, due east

18. Two forces act on an object. One force is 6.0 N horizontally. The second force is 8.0 N vertically.



 a. Find the magnitude and direction of the resultant.

$$F_{R} = \sqrt{(6.0 \text{ N})^{2} + (8.0 \text{ N})^{2}}$$

$$= 10.0 \text{ N}$$

$$\tan \theta_{R} = \frac{8.0}{6.0} = 1.33$$

$$\theta_{R} = 53^{\circ}$$

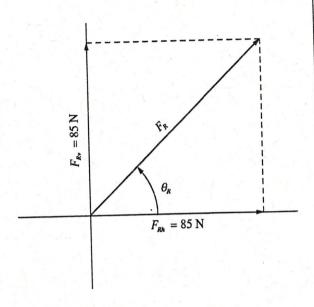
$$F_{R} = 10.0 \text{ N at } 53^{\circ}$$

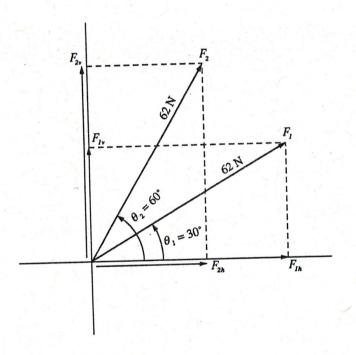
b. If the object is in equilibrium, find the magnitude and direction of the force that produces equilibrium.

$$F_{\rm E} = 10.0 \text{ N at } 53^{\circ} + 180^{\circ} = 233^{\circ}$$

### Practice Problems

19. A 62-N force acts at 30.0° and a second 62-N force acts at 60.0°.





## Practice Problems

a. Determine the resultant force.

Vector addition is most easily carried out by using the method of addition by components. The first step in this method is the resolution of the given vectors into their horizontal and vertical components.

$$F_{1h} = F_1 \cos \theta_1 = (62 \text{ N}) \cos 30^\circ = 54 \text{ N}$$
  
 $F_{1v} = F_1 \sin \theta_1 = (62 \text{ N}) \sin 30^\circ = 31 \text{ N}$ 

$$F_{2h} = F_2 \cos \theta_2 = (62 \text{ N}) \cos 60^\circ = 31 \text{ N}$$
  
 $F_{2v} = F_2 \sin \theta_2 = (62 \text{ N}) \sin 60^\circ = 54 \text{ N}$ 

At this point, the two original vectors have been replaced by four components, vectors that are much easier to add. The horizontal and vertical components of the resultant vector are found by simple addition.

$$F_{Rh} = F_{1r} + F_{2h} = 54 \text{ N} + 31 \text{ N} = 85 \text{ N}$$
  
 $F_{Rv} = F_{1v} + F_{2v} = 31 \text{ N} + 54 \text{ N} = 85 \text{ N}$ 

The magnitude and direction of the resultant vector are found by the usual method.

$$F_{R} = \sqrt{(F_{Rh})^{2} + (F_{Rv})^{2}}$$
$$= \sqrt{(85 \text{ N})^{2} + (85 \text{ N})^{2}}$$
$$= 120 \text{ N}$$

$$\tan \theta_{R} = \frac{F_{RV}}{F_{Rh}} = \frac{85 \text{ N}}{85 \text{ N}} = 1, \ \theta_{R} = 45^{\circ}$$

$$F_{R} = 120 \text{ N at } 45^{\circ}$$

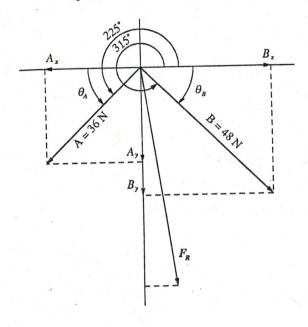
b. What is the magnitude and direction of the force that produces equilibrium?

$$F_{\rm E} = 120 \text{ N}, 225^{\circ}$$

# m

#### Practice Problems

20. Two forces act on an object. A 36-N force acts at 225°. A 48-N force acts at 315°. What would be the magnitude and direction of their equilibrant?



$$\theta_{A} = 225^{\circ} - 180^{\circ} = 45^{\circ}$$

$$\theta_{B} = 360^{\circ} - 315^{\circ} = 45^{\circ}$$

$$A_{x} = -A \cos \theta_{A} = -(36 \text{ N}) \cos 45^{\circ} = -25 \text{ N}$$

$$A_{y} = -A \sin \theta_{A} = (-36 \text{ N}) \sin 45^{\circ} = -25 \text{ N}$$

$$B_{x} = B \cos \theta_{B} = (48 \text{ N}) \cos 45^{\circ} = 34 \text{ N}$$

$$B_{y} = -B \sin \theta_{B} = -(48 \text{ N}) \sin 45^{\circ} = -34 \text{ N}$$

$$F_{x} = A_{x} + B_{x} = -25 \text{ N} + 34 \text{ N} = 9 \text{ N}$$

$$F_{y} = A_{y} + B_{y} = -25 \text{ N} - 34 \text{ N} = -59 \text{ N}$$

$$F_{R} = \sqrt{F_{x}^{2} + F_{y}^{2}} = \sqrt{(-9 \text{ N})^{2} + (-59 \text{ N})^{2}}$$

$$= 60 \text{ N}$$

$$\tan \theta_{R} = \frac{9}{59} = 0.153, \ \theta = 9^{\circ}$$

$$\theta_{R} = 270^{\circ} + 9^{\circ} = 279^{\circ}$$

$$F_{R} = 60 \text{ N} \text{ at } 279^{\circ}$$

 $F_{\rm E} = 60 \text{ N}$  $\theta_{\rm E} = 279^{\circ} - 180^{\circ} = 99^{\circ}$ 

#### **Practice Problems**

21. The sign in the last Example Problem is now hung by ropes that each make an angle of 42° with the horizontal. What force does each rope exert?

Following the method of the Example Problem, the vertical component of the force exerted by each rope must support half of the sign weight.

$$A_{\rm v} = B_{\rm v} = \frac{168 \text{ N}}{2} = 84 \text{ N}$$
  
$$\frac{A_{\rm v}}{\sin 42^{\circ}} = A = \frac{84 \text{ N}}{\sin 42^{\circ}} = 126 \text{ N}$$

$$B = A = 126 \text{ N}$$

22. The people who hung the sign decided to raise it higher by pulling the two ropes more horizontal. They increase the force on each rope to 575 N and keep the angles equal. What angle does each rope make with the horizontal now?

Since 
$$A_v = A \sin \theta$$
, and  $A_v = 84 \text{ N}$   
and  $A = 575 \text{ N}$ ,  
$$\sin \theta = \frac{84}{575} = 0.146$$

$$\theta = 8.4^{\circ}.$$

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- 23. The 562-N trunk is placed on an inclined plane that forms a 66° angle with the horizontal.
  - a. Calculate the values of  $F_{\perp}$  and  $F_{\parallel}$ .

$$F_{\perp} = (562 \text{ N}) \cos 66^{\circ} = 229 \text{ N}$$
  
 $F_{\parallel} = (562 \text{ N}) \sin 66^{\circ} = 513 \text{ N}$ 

b. Compare your results with those given above for the same trunk on a 30° incline.

The perpendicular force is less and the parallel force is greater on the  $66^{\circ}$  incline than in the case of the  $30^{\circ}$  incline.

#### Practice Problems

- 24. A car weighing  $1.2 \times 10^4$  N is parked on a  $36^{\circ}$  slope.
  - a. Find the force tending to cause the car to roll down the hill.

$$F_{\parallel} = (1.2 \times 10^4 \text{ N}) \sin 36^\circ = 7.1 \times 10^3 \text{ N}$$

b. What is the force the car exherts perpendicular to the hill?

$$F_{\perp} = (1.2 \times 10^4 \text{ N}) \cos 36^\circ = 9.7 \times 10^3 \text{ N}$$

- 25. The brakes in the car in Problem 24 fail, and the car starts to roll down the hill. Assume there is no friction.
  - a. What is the acceleration of the car?

$$a = \frac{F}{m} = \frac{F||}{m} = \frac{mg \sin \theta}{m^2}$$

 $g \sin \theta = 5.8 \text{ m/s}^2$ 

b. After it has moved 30 m, how fast is it moving?

$$v^2 = 2ad = 2(5.8 \text{ m/s}^2)(30 \text{ m}), \text{ so}$$
  
 $v = 19 \text{ m/s}$ 

c. Could a sprinter run this fast?

No. A sprinter can run about 10 m/s.

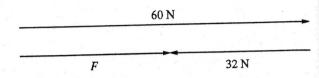
26. The roof on a house rises 1.00 m over a horizontal distance of 3.50 m. A 71.0-kg roofer stands on the roof. Is the frictional force that keeps the roofer from slipping equal in magnitude to  $F_{\perp}$  or  $F_{\parallel}$ ? What is its magnitude?

 $F_{\parallel} = W \sin \theta$ , where we find  $\theta$  from  $\tan \theta = (1.00 \text{ m})/(3.50 \text{ m}) = 0.286$ , so  $\theta = 16.0^{\circ}$   $W = (71.0 \text{ kg})(9.80 \text{ m/s}^2) = 969 \text{ N}$ , so  $F_{\parallel} = 191 \text{ N}$ .

## Chapter Review Problems

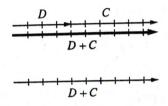
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1. What is the vector sum of a 65-N force acting due east and a 32-N force acting due west?

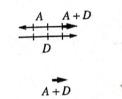


F = 33 N, due east

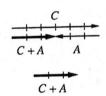
- 2. Graphically find the sum of the following pairs of vectors in Figure 6-25.
  - a. D and C



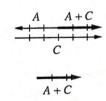
b. A and D



c. C and A



d. A and C



e. E and F



3. a. What is the resultant of a pair of forces, 100 N, upward and 75 N, downward?

25 N, upward

b. What is their resultant if they both act downward?

175 N, downward

- 4. An airplane normally flies at 200 km/h. What is the resultant velocity of the airplane if
  - a. it experiences a 50-km/h tail wind?

Tail wind is in the same direction as the airplane; 200 km/h + 50 km/h = 250 km/h.

b. it experiences a 50-km/h head wind?

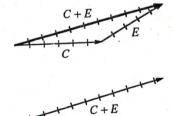
Head wind is in the opposite direction of the airplane; 200 km/h - 50 km/h = 150 km/h

- 5. Graphically add the following pairs of vectors in Figure 6-25.
  - a. B and D



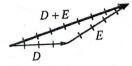


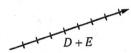
b. C and E



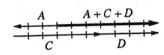
# Chapter Review Problems

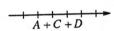
c. D and E



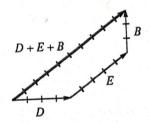


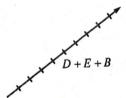
- 6. Graphically add the following vectors in Figure 6–25.
  - a. A + C + D





b. D + E + B



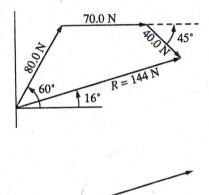


c. B + D + F



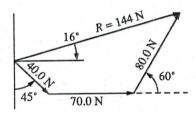
Zero

- 7. Three forces act on point P. Force A has a magnitude of 80.0 N and is directed at  $60.0^{\circ}$ . Force B has a magnitude of 70.0 N and is directed at  $0.0^{\circ}$ . Force C has a magnitude of 40.0 N and is directed at  $315^{\circ}$ .
  - a. Graphically add these three forces in the order A + B + C.



b. Graphically add these three forces in the order C + B + A.

144 N, 16°





c. What is noted about the solutions in each case?

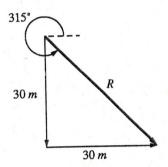
Added head to tail, each arrangement of the vectors should give the same resultant. This is about 144 N at  $16^{\circ}$ .

## Chapter Review Problems

8. You head downstream on a river in a canoe. You can paddle at 5.0 km/h and the river is flowing at 2.0 km/h. How far downstream will you be in 30 minutes?

$$d = vt = (7.0 \text{ km/h})(0.50 \text{ h}) = 3.5 \text{ km}$$

You walk 30 m south and 30 m east. Draw and add vectors for these two displacements. Compute the resultant.



$$R^{2} = A^{2} + B^{2}$$

$$R = \sqrt{(30 \text{ m})^{2} + (30 \text{ m})^{2}}$$

$$= \sqrt{1800 \text{ m}^{2}} = 42 \text{ m}$$

$$\tan \theta = \frac{30 \text{ m}}{30 \text{ m}} = 1$$

$$\theta = 45^{\circ}$$

$$R = 42 \text{ m}, 315^{\circ}$$

10. A ship leaves its home port expecting to travel to a port 500 km due south. Before it can move, a severe storm comes up and blows the ship 100 km due east. How far is the ship from its destination? In what direction must the ship travel to reach its destination?

$$R^{2} = A^{2} + B^{2}$$

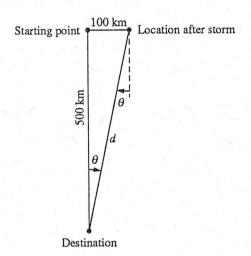
$$R = \sqrt{(100 \text{ km})^{2} + (500 \text{ km})^{2}} = \sqrt{260 000 \text{ km}^{2}}$$

$$= 509 \text{ km}$$

$$\tan \theta = \frac{500 \text{ km}}{100 \text{ km}} = 5$$

$$\theta = 79^{\circ}$$

$$R = 509 \text{ km}, 259^{\circ}$$



11. A hiker leaves camp and, using a compass, walks 4 km E, 6 km S, 3 km E, 5 km N, 10 km W, 8 km N, and 3 km S. At the end of three days, the hiker is lost. By drawing a diagram, compute how far the hiker is from camp and which direction should be taken to get back to camp.

Take north and east to be positive directions.

North: -6 km + 5 km + 8 km - 3 km = 4 km

East: 4 km + 3 km - 10 km = -3 km

The hiker is 4 km north and 3 km west of camp. To return to camp, the hiker must go 3 km east and 4 km south.

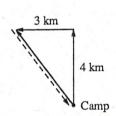
$$R^{2} = A^{2} + B^{2}$$

$$R = \sqrt{(3 \text{ km})^{2} + (4 \text{ km})^{2}} = \sqrt{25 \text{ km}^{2}} = 5 \text{ km}$$

$$\tan \theta = \frac{3 \text{ km}}{4 \text{ km}} = 0.75$$

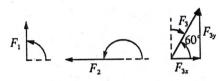
$$\theta = 37^{\circ}$$

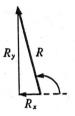
$$R = 5 \text{ km}, 307^{\circ}$$



4 km E + 3 km E + 10 km W = 3 km W6 km S + 5 km N + 8 km + 3 km S = 4 km N

Three forces act simultaneously on point J. One force is 10.0 N north; the second is 15.0 N west; the third is 15.0 N 30.0° east of north. Determine the magnitude and direction of the resultant force.





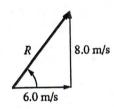
$$F_1 = 10.0 \text{ N north } (90^\circ)$$
  
 $F_2 = 15.0 \text{ N west } (180^\circ)$   
 $F_3 = 15.0 \text{ N } (60^\circ)$ 

$$F_1 = 10.0 \text{ N north } (90^\circ)$$
  $F_{1x} = 10.0 \cos 90 = 0.0$   
 $F_2 = 15.0 \text{ N west } (180^\circ)$   $F_{2x} = 15.0 \cos 180 = -15.0$   
 $F_{3x} = 15.0 \cos 60 = 7.5$ 

$$F_{1y} = 10.0 \sin 90 = 10.0$$
  
 $F_{2y} = 15.0 \sin 180 = 0.0$   
 $F_{3y} = 15.0 \sin 60 = 13.0$ 

$$R_x = 0.0 + (-15.0) + 7.5 = -7.5 \text{ N}$$
  
 $R_y = 10.0 + 0.0 + 13.0 = 23.0 \text{ N}$   
 $R^2 = \sqrt{R^2_x + R^2_y} = \sqrt{(-7.5 \text{ N})^2 + (23.0 \text{ N})^2} = \sqrt{585.3 \text{ N}^2} = 24$   
 $\tan \theta = \frac{7.5}{23.0} = 0.326$   
 $\theta = 18^\circ$   
 $R = 24.2 \text{ N}, 108^\circ$ 

Diane rows a boat at 8.0 m/s directly across a river that flows at 6.0 m/s.



a. What is the resultant speed of the boat?

$$R^2 = A^2 + B^2$$
  
 $R = \sqrt{(8.0 \text{ m/s})^2 + (6.0 \text{ m/s}^2)} = \sqrt{100 \text{ m}^2/\text{s}^2} = 10 \text{ m/s}$   
 $\tan \theta = \frac{8.0 \text{ m/s}}{6.0 \text{ m/s}} = 1.33$   
 $\theta = 53^\circ$   
 $R = 10 \text{ m/s}, 53^\circ$ , as measured from the bank

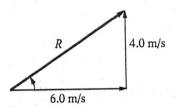
b. If the stream is 240 m wide, how long will it take Diane to row across?

$$v = \frac{d}{t}$$
, so  $t = \frac{d}{v} = \frac{240 \text{ m}}{8.0 \text{ m/s}} = 30 \text{ s}$ 

c. How far downstream will Diane be?

$$d = vt = (6.0 \text{ m/s})(30) = 1.8 \times 10^2 \text{ m}$$

14. Dave rows a boat across a river at 4.0 m/s. The river is flowing at 6.0 m/s and it is 360 m across.



a. In what direction, relative to the shore, does Dave's boat go?

$$\tan \theta = \frac{4.0}{6.0} = 0.67$$
 $\theta = 34^{\circ}$ 

b. How long does it take Dave to cross the river?

$$v = \frac{d}{t}$$
, so  $t = \frac{d}{v} = \frac{360 \text{ m}}{4.0 \text{ m/s}} = 90 \text{ s}$ 

c. How far downstream is Dave's landing point?

$$d = vt = 6.0(90) = 5.4 \times 10^2 \text{ m}$$

d. How long would it take Dave to cross the river if there were no current?

90 s

15. Kyle is flying a plane due north at 225 km/h as a wind carries it due east at 55 km/h. Find the magnitude and direction of the plane's resultant velocity analytically.

$$R^{2} = A^{2} + B^{2}$$

$$v = \sqrt{(55 \text{ km/h})^{2}} + (225 \text{ km/h})^{2}$$

$$= 230 \text{ km/h}$$

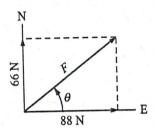
$$\tan \theta = \frac{225 \text{ km/h}}{55 \text{ km/h}} = 4.09$$

$$\theta = 76^{\circ}$$

$$v = 230 \text{ km/h}, 76^{\circ}$$

### Chapter Review Problems

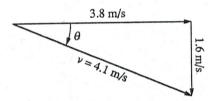
16. Sue and Jenny kick a soccer ball at exactly the same time. Sue's foot exerts a force of 66 N north. Jenny's foot exerts a force of 88 N east. What is the magnitude and direction of the resultant force on the ball?



$$R^2 = A^2 + B^2$$
  
 $R = \sqrt{(66 \text{ N})^2 + (88 \text{ N})^2} = \sqrt{12 \ 100 \text{ N}^2} = 110 \text{ N}$   
 $\tan \theta = \frac{66 \text{ N}}{88 \text{ N}} = 0.75$ 

$$\theta = 37^{\circ}$$
  
 $F = 1.1 \times 10^{2} \text{ N}, 37^{\circ}$ 

17. Kym is in a boat traveling 3.8 m/s straight across a river 240 m wide. The river is flowing at 1.6 m/s.



a. What is Kym's resultant velocity?

$$R^2 = A^2 + B^2$$
  
 $R = \sqrt{(3.8 \text{ m/s})^2 + (1.6 \text{ m/s})^2} = 4.1 \text{ m/s}$   
 $\tan \theta = \frac{3.8 \text{ m/s}}{1.6 \text{ m/s}} = 2.38$   
 $\theta = 67^\circ \text{ from bank}$ 

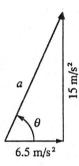
b. How long does it take Kym to cross the river?

$$v = \frac{d}{t}$$
, so  $t = \frac{d}{v} = \frac{2.40 \text{ m}}{3.8 \text{ m/s}} = 63 \text{ s}$ 

c. How far is Kym downstream when Kym reaches the other side?

$$d = vt = (1.6 \text{ m/s})(63 \text{ s}) = 1.0 \times 10^2 \text{ m}$$

18. A weather station releases a weather balloon. The balloon's buoyancy accelerates it straight up at 15 m/s². At the same time, a wind accelerates it horizontally at 6.5 m/s². What is the magnitude and direction (with reference to the horizontal) of the resultant acceleration?

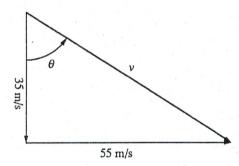


$$a = \sqrt{(6.5 \text{ m/s}^2)^2 + (15 \text{ m/s}^2)^2} = 16 \text{ m/s}^2$$

$$\tan \theta = \frac{15 \text{ m/s}^2}{6.5 \text{ m/s}^2} = 2.31$$

$$\theta = 67^{\circ}$$
  
 $a = 16 \text{ m/s}^2, 67^{\circ}$ 

19. A descent vehicle landing on the moon has a vertical velocity toward the surface of the moon of 35 m/s. At the same time, it has a horizontal velocity of 55 m/s.



a. At what speed does the vehicle move along its descent path?

$$R^2 = A^2 + B^2$$
  
 $R = \sqrt{(55 \text{ m/s})^2 + (35 \text{ m/s})^2} = 65 \text{ m/s}$ 

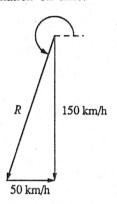
b. At what angle with the vertical is this path?

$$\tan \theta = \frac{55 \text{ m/s}}{35 \text{ m/s}} = 1.57$$

$$\theta = 58^{\circ}$$

#### Chapter Review Problems

20. Kyle wishes to fly to a point 450 km due south in 3.00 hours. A wind is blowing from the west at 50 km/h. Compute the proper heading and speed that Kyle must choose in order to reach his destination on time.



With no wind, 
$$v = \frac{d}{t} = \frac{450 \text{ km}}{3.00 \text{ h}} = 150 \text{ km/h}$$

Kyle's velocity and the wind must add to 150 km/h

$$R^2 = A^2 + B^2$$

$$R = \sqrt{(50 \text{ km/h})^2 + (150 \text{ km/h})^2}$$
  
= 158 km/h

$$\tan \theta = \frac{50 \text{ km/h}}{150 \text{ km/h}} = 0.333$$

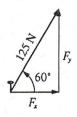
$$9 = 18^{\circ}$$

$$R = 158$$
 km/h,  $18^{\circ}$  west of south

21. Dan applies a force of 92 N on a heavy box by using a rope held at an angle of 45° with the horizontal. What are the vertical and horizontal components of the 92-N force?

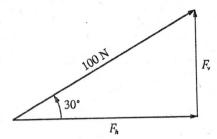
$$F_h = F \cos \theta = (92 \text{ N}) \cos 45 = 65 \text{ N}$$
  
 $F_v = F \sin \theta = (92 \text{ N}) \sin 45 = 65 \text{ N}$ 

22. Beth, a construction worker, attempts to pull a stake out of the ground by pulling on a rope that is attached to the stake. The rope makes an angle of 60.0° with the horizontal. Beth exerts a force of 125 N on the rope. What is the magnitude of the upward component of the force acting on the stake?



$$F_v = F \sin \theta = (125 \text{ N}) \sin 60 = 108 \text{ N}$$

23. A 40-kg crate is pulled across the ice with a rope. A force of 100 N is applied at an angle of 30° with the horizontal.



Calculate

a. the acceleration of the crate.

$$F_h = F \cos \theta$$
  
 $a = \frac{F_h}{m} = \frac{F \cos \theta}{m} = \frac{(100 \text{ N}) \cos 30}{40 \text{ kg}} = 2.2 \text{ m/s}^2$ 

b. the upward force the ice exerts on the crate as it is pulled. Neglect friction.

$$F_v = F \sin \theta = (100 \text{ N}) \sin 30 = 50 \text{ N}, \text{ up}$$
  
 $W = mg = (40 \text{ kg})(9.80 \text{ m/s}^2) = 390 \text{ N}, \text{ down}$   
 $F_{\text{ice}} = W - F_v = 390 \text{ N} - 50 \text{ N} = 340 \text{ N}, \text{ up}$ 

24. Joe pushes on the handle of a 10-kg lawn spreader. The handle makes a 45° angle with the horizontal. Joe wishes to accelerate the spreader from rest to 1.39 m/s in 1.5 s. What force must Joe apply to the handle? Neglect friction.

$$a = \frac{v}{t}$$
 and  $F_h = ma = F \cos \theta$  so,  

$$F = \frac{ma}{\cos \theta} = \frac{mv}{t \cos \theta} = \frac{(10 \text{ kg})(1.39 \text{ m/s})}{(1.5 \text{ s})(\cos 45)} = 13 \text{ N}$$

25. Tammy leaves the office, drives 26 km due north, then turns onto a second highway and continues in a direction of 30.0° north of east for 62 km. What is her total displacement from the office? Add displacements by components.

$$d_1 = 26 \text{ km}$$
, north  $d_{1h} = (26 \text{ km}) \cos 90 = 0$   $d_{1v} = (26 \text{ km}) \sin 90 = 26 \text{ km}$   $d_2 = 62 \text{ km}$ ,  $30.0^{\circ}$   $d_{2h} = (62 \text{ km}) \cos 30 = 54 \text{ km}$   $d_{2v} = (62 \text{ km}) \sin 30 = 31 \text{ km}$ 

$$R_h = 0 + 54 \text{ km} = 54 \text{ km}$$
 $R_v = 26 \text{ km} + 31 \text{ km} = 57 \text{ km}$ 
 $R^2 = R_h^2 + R_v^2$ 

$$R = \sqrt{(54 \text{ km})^2 + (57 \text{ km})^2} = 79 \text{ km}$$
 $\tan \theta = \frac{57 \text{ km}}{54 \text{ km}} = 1.056,$ 
so  $\theta = 47^\circ$ 
 $R = 79 \text{ km}, 47^\circ$ 

- 26. Find the magnitude of the resultant of a 40-N force and a 70-N force acting concurrently when the angle between them is
  - a. 0.0°

$$40 N + 70 N = 110 N$$

b. 30.0°

$$R_h = 40 \text{ N} + 70 \cos 30 = 10 \text{ N}$$
  $R_v = 70 \sin 30 = 35 \text{ N}$   $R = \sqrt{(101 \text{ N})^2 + (35 \text{ N})^2} = 107 \text{ N}$ 

c. 60.0°

$$R_h = 40 \text{ N} + 70 \cos 60 = 75 \text{ N}$$
  $R_v = 70 \sin 60 = 61 \text{ N}$   
 $R = \sqrt{(75 \text{ N})^2 + (70 \text{ N})^2} = 97 \text{ N}$ 

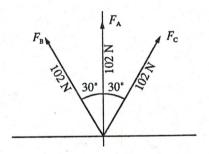
**d.** 90.0°

$$R_h = 40 \text{ N}$$
  $R_v = 70 \text{ N}$   
 $R = \sqrt{(40 \text{ N})^2 + (70 \text{ N})^2} = 81 \text{ N}$ 

e. 180.0°

$$70 N - 40 N = +30 N$$

27. Three people attempt to haul a heavy sign to the roof of a building by using three ropes attached to the sign. Abby stands directly above the sign and pulls straight up on a rope. Eric and Kim stand on either side of Abby. Their ropes form 30.0° angles with Abby's rope. A force of 102 N is applied on each rope. What is the net upward force acting on the sign? See Figure 6-26.



Eric: 
$$F_{\text{eh}} = (102 \text{ N}) \cos 60.0 = -51.0 \text{ N}$$
  
Abby:  $F_{\text{ah}} = (102 \text{ N}) \cos 90.0 = 0.0$   
Kim:  $F_{\text{kh}} = (102 \text{ N}) \cos 60.0 = 51.0 \text{ N}$   
 $R_{\text{h}} = -51.0 \text{ N} + 0 + 51.0 \text{ N} = 0$ 

$$R_h = -51.0 \text{ N} + 0 + 51.0 \text{ N} = 0$$
  
 $R_v = 88.3 \text{ N} + 102 \text{ N} + 88.3 \text{ N} = 279 \text{ N}, \text{ up}$ 

$$F_{\text{ev}} = (102 \text{ N}) \sin 60.0 = 88.3 \text{ N}$$
  
 $F_{\text{ah}} = (102 \text{ N}) \sin 90.0 = 102 \text{ N}$   
 $F_{\text{kv}} = (102 \text{ N}) \sin 60.0 = 88.3 \text{ N}$ 

- 28. A river flows toward 90°. Mark, a riverboat pilot, heads the boat at 297° and is able to go straight across the river at 6.0 m/s.

  See Figure 6-27.
  - a. What is the velocity of the current?

$$\tan \theta = \frac{v_c}{6.0 \text{ m/s}}$$
, so  $v_c = (6.0 \text{ m/s}) \tan 63^\circ = 12 \text{ m/s}$ ,  $90^\circ$ 

b. What is the velocity of the boat as seen from the river bank?

6.0 m/s, 0°

Note:  $v_c$  is the velocity of the boat in still water.

29. An object in equilibrium has three forces exerted on it. A 33-N force acts at 90°, and a 44-N force acts at 60°. What is the magnitude and direction of the third force? Solution by components.

$$F_1 = 33 \text{ N}, 90^{\circ}$$
  $F_{1h} = (33 \text{ N}) \cos 90 = 0$   $F_{1v} = (33 \text{ N}) \sin 90 = 33 \text{ N}$   
 $F_2 = 44 \text{ N}, 60^{\circ}$   $F_{2h} = (44 \text{ N}) \cos 60 = 22 \text{ N}$   $F_{2v} = (44 \text{ N}) \sin 60 = 38 \text{ N}$   
 $F_3 = ?$   $F_{3h} = x$   $F_{3v} = y$ 

For equilibrium, the sum of the components must equal zero, so

$$0 + 22 \text{ N} + x = 0 \text{ and } x = -22 \text{ N}$$

$$33 \text{ N} + 38 \text{ N} + y = 0 \text{ and } y = -71 \text{ N}$$

$$R = \sqrt{(-22 \text{ N})^2 + (-71 \text{ N})^2} = 74 \text{ N}$$

$$\tan \theta = \frac{-71 \text{ N}}{-22 \text{ N}} = 3.23, \text{ so } \theta = 73^\circ$$

$$F_3 = 74 \text{ N}, 253^\circ$$

30. Five forces act on an object: the first, 60 N at 90°; the second, 40 N at 0°; the third, 80 N at 270°; the fourth, 40 N at 180°; and the fifth, 50 N at 60°. What is the magnitude and direction of a sixth force that produces equilibrium of the object?

Solutions by components

$$F_1 = 60 \text{ N}, 90 ^{\circ}$$
  $F_{1h} = (60 \text{ N}) \cos 90 = 0$   $F_{1v} = (60 \text{ N}) \sin 90 = 60 \text{ N}$   
 $F_2 = 40 \text{ N}, 0^{\circ}$   $F_{2h} = (40 \text{ N}) \cos 0 = 40 \text{ N}$   $F_{2v} = (40 \text{ N}) \sin 0 = 0$   
 $F_3 = 80 \text{ N}, 270^{\circ}$   $F_{3h} = (80 \text{ N}) \cos 270 = 0$   $F_{3v} = (80 \text{ N}) \sin 270 = -80 \text{ N}$   
 $F_4 = 40 \text{ N}, 180^{\circ}$   $F_{4h} = (40 \text{ N}) \cos 180 = -40 \text{ N}$   $F_{4v} = (40 \text{ N}) \sin 180 = 0$   
 $F_5 = 50 \text{ N}, 60^{\circ}$   $F_{5h} = (50 \text{ N}) \cos 60 = 25 \text{ N}$   $F_{5v} = (50 \text{ N}) \sin 60 = 43 \text{ N}$   
 $F_6 = ?$   $F_{6h} = x$   $F_{6v} = y$ 

$$0 + 40 \text{ N} + 0 + (-40 \text{ N}) + 25 \text{ N} + x = 0$$
, so  $x = -25 \text{ N}$   
 $60 \text{ N} + 0 + (-80 \text{ N}) + 0 + 43 \text{ N} + y = 0$ , so  $y = -23 \text{ N}$ 

$$R = \sqrt{(-25 \text{ N})^2 + (-23 \text{ N})^2} = 34 \text{ N}$$
  

$$\tan \theta = \frac{-23 \text{ N}}{-25 \text{ N}} = 0.92, \text{ so } \theta = 43^{\circ}$$
  

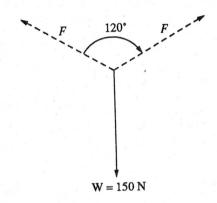
$$F_6 = 34 \text{ N}, 223^{\circ}$$

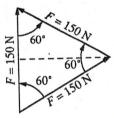
79

31. A street lamp weighs 150 N. It is supported equally by two wires that form an angle of 120° with each other.

Solution by components.

a. What is the tension of these wires?

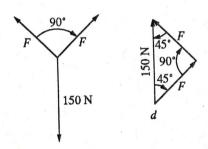




Horizontal  $-T_{1h} + T_{2h} = 0$ , so  $T_{1h} = T_{1v}$  and  $T_1 \cos 30 = T_2 \cos 30$ , so  $T_1 = T_2$ Vertical,  $T_{1v} + T_{2v} - 150 \text{ N} = 0$ , so  $T_1 \sin 30 + T_1 \sin 30 = 150$ 

$$T_1 = T_2 = \frac{150}{2 \sin 30} = 150 \text{ N}$$

b. If the angle between the wires is reduced to 90.0°, what new force does each wire exert?



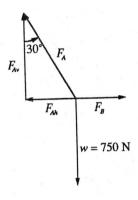
$$T_1 = T_2 = \frac{150}{2 \sin 45} = 106 \text{ N}$$

## Chapter Review Problems

c. As the angle between the wire decreases, what happens to the force on the wire?

decreases toward 75 N

Joe wishes to hang a sign weighing 750 N so 32. that cable A attached to the store makes a 30° angle as shown in Figure 6-28. Cable B is attached to an adjoining building. Calculate the necessary tension in cable B.



Solution by components.

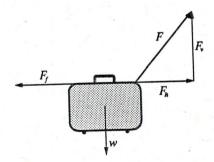
The sum of the components must equal zero, so  $F_{Av} - W = 0$  and  $F_{Av} = F \sin 60.0 = 750 \text{ N}$ 

$$F = \frac{750 \text{ N}}{\sin 60} = 866 \text{ N}$$

Also,  $F_B - F_{Ah} = 0$ , so  $F_B - F_{Ah} = (866 \text{ N}) \cos 60 = 433 \text{ N}$ , right

Rachel pulls her 18-kg suitcase at a constant speed by pulling on a handle that makes an The frictional angle  $\theta$  with the horizontal. force on the suitcase is 27 N and Rachel exerts a 43-N force on the handle.

> Rachel pulls at constant speed, so the suitcase is at equilibrium and the sum of components equals zero.



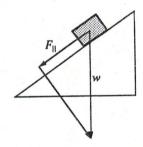
a. What angle does the handle make with the horizontal?

$$F_{\rm h} - F_{\rm f} = 0$$
, so  $F_{\rm h} = F_{\rm f} = 27$  N and   
 $\cos \theta = \frac{F_{\rm h}}{F} = \frac{27 \text{ N}}{43 \text{ N}}$ , so  $\theta = 51^{\circ}$ .

**b.** What is the normal force exerted on the suitcase?

$$F_{\rm N} + F_{\rm V} - W = 0$$
, so  
 $F_{\rm N} = W - F_{\rm V} = mg - F \sin \theta$   
= (18 kg)(9.80 m/s<sup>2</sup>) - 43 sin 51°  
= 1.4 × 10<sup>2</sup> N, up

34. You place a box weighing 215 N on an inclined plane that makes a 35.0° angle with the horizontal. Compute the component of the gravitational force acting down the inclined plane.



$$F_{\parallel} = W \sin \theta$$
  
= (215 N) sin 35°  
= 123 N

- 35. You slide a 325-N trunk up a 20.0° inclined plane with a constant velocity by exerting a force of 211 N parallel to the inclined plane.
  - a. What is the component of the trunk's weight parallel to the plane?

$$F_{\parallel} = W \sin \theta = (325 \text{ N}) \sin 20 = 111 \text{ N}$$

b. What is the sum of your applied force, friction, and the parallel component of the trunk's weight? Why?

zero, because the velocity is constant

# c. What is the size and direction of the friction force?

Let up plane be positive, then 
$$F - F_{\parallel} - F_{\rm f} = 0$$
  
211 N - 111 N -  $F_{\rm f} = 0$ , so  $F_{\rm f} = 100$  N, downward along the plane

d. What is the coefficient of friction?

$$\mu = \frac{F_f}{F_N} = \frac{100 \text{ N}}{325 \cos 20} = 0.327$$

36. What force would you have to exert on the trunk in Problem 35 so that it would side down the plane with a constant velocity? What would be the direction of the force? Positive direction still up plane.

$$F - F_{||} + F_{f} = 0$$
, so  
 $F = F_{||} - F_{f} = W \sin \theta - \mu W \cos \theta$   
= (325 N) sin 20  
- (0.327)(325) cos 20  
= 11 N, up plane

- 37. A 2.5-kg block slides down a 25° inclined plane with constant acceleration. The block starts from rest at the top. At the bottom, its velocity reaches 0.65 m/s. The length of the incline is 1.6 m.
  - a. What is the acceleration of the block?

$$v^2 = v^2_0 + 2ad$$
, but  $v_0 = 0$ ,  
so  $a = \frac{v^2}{2d} = \frac{(0.65 \text{ m/s})^2}{2(1.6 \text{ m})} = 0.13 \text{ m/s}^2$ 

**b.** What is the coefficient of friction between the plane and block?

Let up plane be positive. Then, 
$$F_f - F_{\parallel} = -(ma)$$
, so  $Ff = F_{\parallel} - (ma)$ 

$$\mu = \frac{F_f}{F_N} = \frac{F_{\parallel} - (ma)}{F_N} = \frac{W \sin \theta - (ma)}{W \cos \theta}$$

$$= \frac{mg \sin \theta - ma}{mg \cos \theta} = \frac{m(g \sin \theta - a)}{mg \cos \theta}$$

$$= \frac{(9.80 \text{ m/s}^2) \sin 25 - (0.13 \text{ m/s}^2)}{(9.80 \text{ m/s}^2) \cos 25}$$

$$= \frac{4.01 \text{ m/s}^2}{8.88 \text{ m/s}^2}$$

$$= 0.452$$

c. Does the result of either a or b depend on the mass of the block?

No. In part b, the mass divides out.

# Supplemental Problems (Appendix B)

1. Find  $\theta$  if

**a.** 
$$\tan \theta = 9.5143$$
.  $\theta = 84.000^{\circ}$ 

**b.** 
$$\sin \theta = .4540$$
.  $\theta = 27.00^{\circ}$ 

**c.** 
$$\cos \theta = .8192$$
.  $\theta = 35.00^{\circ}$ 

**d.** 
$$\tan \theta = .1405$$
.  $\theta = 7.998^{\circ}$ 

e. 
$$\sin \theta = .7547$$
.  $\theta = 49.00^{\circ}$ 

f. 
$$\cos \theta = .9781$$
.  $\theta = 12.01^{\circ}$ 

2. Find the value of:

a. 
$$\tan 28^{\circ} = 0.53$$

**b.** 
$$\sin 86^{\circ} = 1.0$$

c. 
$$\cos 2^{\circ} = 1$$

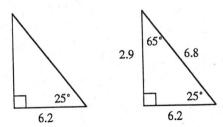
**d.** 
$$\tan 58^{\circ} = 1.6$$

e. 
$$\sin 40^{\circ} = 0.64$$

f. 
$$\cos 71^{\circ} = 0.33$$

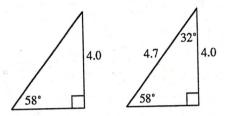
3. Solve for all sides and all angles for the following right triangles.

a.

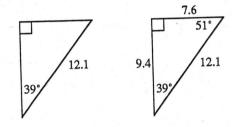


## Supplemental Problems (Appendix B)

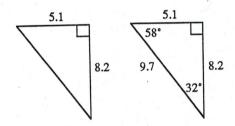
b.



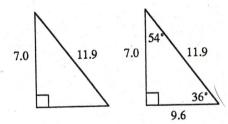
c.



d.



e.



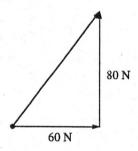
- An 80-N and a 60-N force act concurrently on a point. Find the magnitude of the vector sum if the forces pull
  - a. in the same direction.

b. in opposite directions.



20 N

c. at a right angle to each other.



100 N

5. You head downstream on a river in an outboard. The current is flowing at a rate of 1.50 m/s. After 30.0 min, you find that you have traveled 24.3 km. How long will it take you to trave back upstream to your original point of departure?

Downstream:

$$v_{\text{down}} = \frac{(24.3 \text{ km}) (1.00 \times 10^3 \text{ m/km})}{(30.0 \text{ min})(60.0 \text{ s/m i n})}$$

$$= 13.5 \text{ m/s}$$

$$v_{\text{boat}} = v_{\text{down}} - v_{\text{current}} = 13.5 \text{ m/s} - 1.50 \text{ m/s}$$

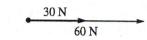
$$v_{\text{boat}} = 12.0 \text{ m/s}$$

Upstream:

$$v_{\rm up} = v_{\rm boat} - v_{\rm current} = 12.0 \text{ m/s} - 1.50 \text{ m/s}$$
  
= 10.5 m/s  
 $v_{\rm up} = d/t$ ; so  $t = d/v_{\rm up} = 2.43 \times 10^4 \text{ m/10.5 m/s}$   
 $t = 2.31 \times 10^3 \text{ s} = 38.5 \text{ min}$ 

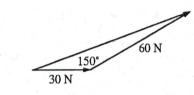
6. One force of 60 N and a second of 30 N act on an object at point P. Graphically add the vectors and find the magnitude of the resultant when the angle between them is as follows.

a. 0°



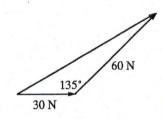
90 N

**b.** 30°



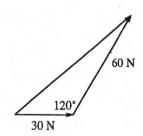
87 N

c. 45°



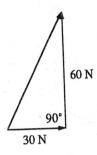
84 N

d. 60°



79 N

e. 90°



67 N

f. 180°



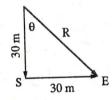
30 N

7. You walk 30 m south and 30 m east. Draw and add vectors representing these two displacements.

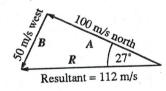
$$R^2 = (30 \text{ m})^2 + (30 \text{ m})^2$$
  
 $R = 42 \text{ m}, 315^\circ$ 

$$\tan \theta = \frac{30 \text{ m}}{30 \text{ m}} = 1$$

$$\theta = 45^{\circ}$$



8. A plane flying at  $90^{\circ}$  at  $1.00 \times 10^{2}$  m/s is blown toward  $180^{\circ}$  at  $5.0 \times 10^{1}$  m/s by a strong wind. Find the plane's resultant velocity and direction.



 $v = 112 \text{ m/s}, 117^{\circ}$ 

# Supplemental Problems

9. In tackling a running back from the opposing team, a defensive lineman exerts a force of 500 N at 180°, while a linebacker simultaneously applies a force of 650 N at 270°. What is the resultant force on the ball carrier?

$$R^{2} = 650^{2} + 500^{2}$$

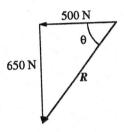
$$R = 820 \text{ N}$$

$$\tan \theta = \frac{650}{500}$$

$$\theta = 52^{\circ}$$

$$180^{\circ} + 52 = 232^{\circ}$$

$$F = 820 \text{ N}, 232^{\circ}$$



10. A hobo hops a freight car 15 m long and 3.0 m wide. The car is moving east at 2.5 m/s. Exploring the surroundings, the hobo walks from corner A to corner B in 20.0 s; then from corner B to corner C in 5.0 s as shown below. With the aid of a vector diagram, compute the hobo's displacement relative to the ground.

$$d_{car} = vt = (2.5 \text{ m/s})(25.0 \text{ s}) = 63 \text{ m}$$

$$d_{E} = 63 \text{ m} - 15 \text{ m} = 48 \text{ m}$$

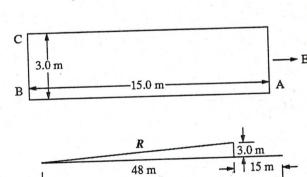
$$R^{2} = (48 \text{ m})^{2} + (3.0 \text{ m})^{2}$$

$$R = 48.1 \text{ m}$$

$$R = 48.1 \text{ m}, 3.6^{\circ} \text{ N of E}$$

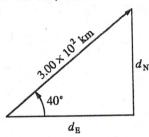
$$\tan \theta = \frac{3.0 \text{ m}}{48} = 0.063$$

$$\theta = 3.6^{\circ}$$



11. A plane travels on a heading of 40.0° for a distance of 3.00 × 10² km. How far north and how far east does the plane travel?

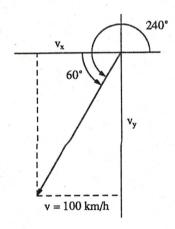
$$d_{\rm N} = d \sin \theta = (3.00 \times 10^2 \text{ km})(\sin 40^{\circ})$$
  
= 1.93 × 10<sup>2</sup> km, north  
 $d_{\rm B} = d \cos \theta = (3.00 \times 10^2 \text{ km})(\cos 40^{\circ})$   
= 2.30 × 10<sup>2</sup> km, east



12. A water skier is towed by a speedboat. The skier moves to one side of the boat in such a way that the tow rope forms an angle 55° with the direction of the boat. The tension on the rope is 350 N. What would be the tension on the rope if the skier were directly behind the boat?

$$F_{\rm T} = (350 \text{ N})(\cos 55^{\circ}) = 200 \text{ N}$$

13. What are the x and y components of a velocity vector of magnitude 100 km/h and direction of  $240^{\circ}$ .



$$v_x = v \cos \theta = (100 \text{ km/h}) \cos 240^\circ$$
  
= - (100 km/h cos 60° = -50.0 km/h  
 $v_y = v \sin \theta = (100 \text{ km/h}) \sin 240^\circ$   
= - (100 km/h) sin 60° = -86.6 km/h  
 $v_x = -50.0 \text{ km/h}, v_y = -86.6 \text{ km/h}$ 

### Supplemental Problems

- 14. Wendy pushes a lawn spreader across a lawn by applying a force of 95 N along the handle that makes an angle of 60.0° with the horizontal.
  - a. What are the horizontal and vertical components of the force?

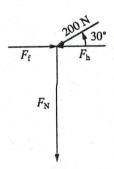
$$F_h = F \cos \theta = (95 \text{ N})(\cos 60^\circ) = 48 \text{ N}$$
  
 $F_v = F \sin \theta = (95 \text{ N})(\sin 60^\circ) = 82 \text{ N}$ 

b. The handle is lowered so it makes an angle of 30.0° with the horizontal. Now what are the horizontal and vertical components of the force?

$$F_h = F \cos \theta = (95 \text{ N})(\cos 30^\circ) = 82 \text{ N}$$
  
 $F_v = F \sin \theta = (95 \text{ N})(\sin 30^\circ) = 48 \text{ N}$ 

15. A brick layer applies a force of 100 N along each of two handles of a wheelbarrow. Its mass is 20 kg and it is loaded with 30 bricks, each of mass 1.5 kg. The handles of the wheelbarrow are 30° from the horizontal and the coefficient of friction is 0.20. What initial acceleration is given the wheelbarrow?

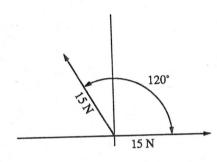
$$F_{\rm N} = \pm (200 \text{ N}) \sin 30^{\circ} [20 \text{ kg} + (1.5 \text{ kg} \times 30)](9.8 \text{ m/s}^2)$$
  
= 637 N  
 $F_{\rm f} = \mu F_{\rm N} = (0.20)(637 \text{ N}) = 127 \text{ N}$   
 $F_{\rm h} = 200 \text{ N} \cos 30^{\circ} = 173 \text{ N}$   
 $a = F/m = (173 \text{ N} - 127 \text{ N})/65 \text{ kg}$   
= 0.71 m/s<sup>2</sup>



Two 15-N forces act concurrently on point P. Find the magnitude of their resultant when the angle between them is

See solution to Problem 4 for notation and method of solution using vector addition by components.

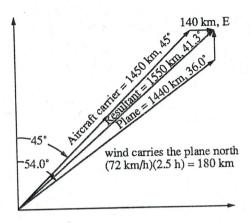
- a. 0.0°. 30 N
- b. 30.0°. 29 N
- c. 90.0°. 21 N
- d. 120.0°.



$$F_h = 15 \text{ N} + (15 \text{ N}) \cos 120^{\circ}$$
  
= 15 N - 7.5 N = 7.5 N  
 $F_v = (15 \text{ N}) \sin 120^{\circ} = 13 \text{ N}$   
 $F = \sqrt{(7.5 \text{ N})^2 + (13 \text{ N})^2} = 15 \text{ N}$ 

- e. 180.0°. 0 N
- 17. You are a pilot on an aircraft carrier. must fly to another aircraft carrier, now 1450 km at 45° of your position, moving at 56 km/h due east. The wind is blowing from the south at 72 km/h. Calculate the heading and air speed needed to reach the carrier 2.5 h after you take off. Hint: Draw a displacement vector diagram.

## Supplemental Problems



The position of second carrier in 2.5 h (56 km/h)(2.5 h) = 140 km, E $R^2 = 1450 \text{ km}, 45^\circ + 140 \text{ km}, 0^\circ$  $= 1550 \text{ km}, 41.3^{\circ}$ 

The wind will carry the plane 180 km, north during the 2.5 h.

Therefore,

 $d_1 + 180$  km,  $90^\circ = R^2 = 1550$  km,  $41.3^\circ$  $d_1 = 1440$  km,  $36.0^{\circ}$ Heading =  $90.0^{\circ} - 36.0^{\circ} = 54.0^{\circ}$  E of N

Air speed = 
$$\frac{1440 \text{ km}}{2.5 \text{ h}}$$
 = 580 km/h

580 km/h, 54° E of N

A 33-N force acting at 90° and a 44-N force acting at 60° act concurrently on point P. What is the magnitude and direction of a third force that produces equilibrium at point P?

Solution by component method.

$$F_{1h} = 0 \text{ N}$$

$$F_{1v} = F_1 = 33 \text{ N}$$

$$F_{2h} = (44 \text{ N}) \cos 60^{\circ} = 22 \text{ N}$$
  
 $F_{2v} = (44 \text{ N}) \sin 50^{\circ} = 38 \text{ N}$ 

$$F_{\rm h} = 0 \text{ N} + 22 \text{ N} = 22 \text{ N}$$
  
 $F_{\rm v} = 33 \text{ N} + 38 \text{ N} = 71 \text{ N}$ 

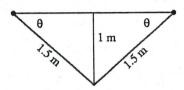
The resultant of the two given forces is

$$F = \sqrt{(22 \text{ N})^2 + (71 \text{ N})^2} = 74 \text{ N}$$

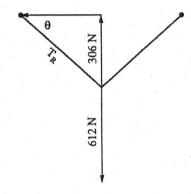
$$\tan \theta = \frac{71 \text{ N}}{22 \text{ N}} = 3.23, \ \theta = 73^{\circ}$$

Equilibrant: 74 N, 253°

19. A person weighs 612 N. If the person sits in the middle of a hammock that is 3.0 m long and sags 1.0 m below the points of support, what force wouldbe exerted by each of the two hammock ropes?



$$\sin \theta = \frac{1}{1.5} = 0.667$$



$$\sin \theta = \frac{306 \text{ N}}{T_{\text{R}}}$$

$$T_{\rm R} = \frac{306 \text{ N}}{\sin \theta} = \frac{306 \text{ N}}{0.667} = 460 \text{ N}$$

20. A bell ringer decides to use a bowling ball to ring the bell. He hangs the 7.3-kg ball from the end of a 2.0 m long rope. He attaches another rope to the ball, and pulls it horizontally until the ball has moved 0.60 m away from the vertical to pull the ball back. How much force must he apply?

Angle pulled back from vertical given by  $\sin \theta = 0.60/2.0 = 0.30$ .

In equilibrium forces balance.

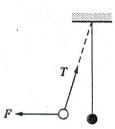
Vertical:  $T \cos \theta = mg$ ;

horizontal:  $T \sin \theta = F$ , where T is the tension in the 2.0 m rope and F is the force on the horizontal rope.

Thus 
$$F/mg = \tan \theta$$
 so

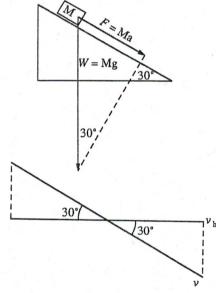
$$F = (7.3 \text{ kg})(9.8 \text{ m/s}^2)(0.31) = 22 \text{ N}$$

#### Supplemental Problems



- 21. A mass, M, starts from rest and slides down the frictionless incline as shown. As it leaves the incline, its speed is 24 m/s. Note: The angle of the incline is 30.0°.
  - **a.** What is the acceleration of the mass while on the incline?

$$F = W(\sin 30.0^{\circ}) = 0.5 W$$
  
 $Ma = 0.5 Mg$   
 $a = 0.5 g = 4.9 \text{ m/s}^2$ 



b. What is the length of the incline?

$$v_{\rm f}^2 = v_{\rm i}^2 + 2ad$$

$$d = \frac{v_{\rm f}^2 - v_{\rm i}^2}{2a} = \frac{(24 \text{ m/s})^2 - 0}{(2)(4.9 \text{ m/s}^2)}$$

$$d = \frac{576 \text{ m}^2/\text{s}^2}{(2)4.9 \text{ m/s}^2} = 59 \text{ m}$$

c. How long does it take the mass to reach the floor after it leaves the top of the incline?

$$a = \frac{\Delta v}{\Delta t}$$
, or  $\Delta t = \frac{\Delta v}{a} = \frac{24 \text{ m/s}}{4.9 \text{ m/s}^2} = 4.9 \text{ s}$